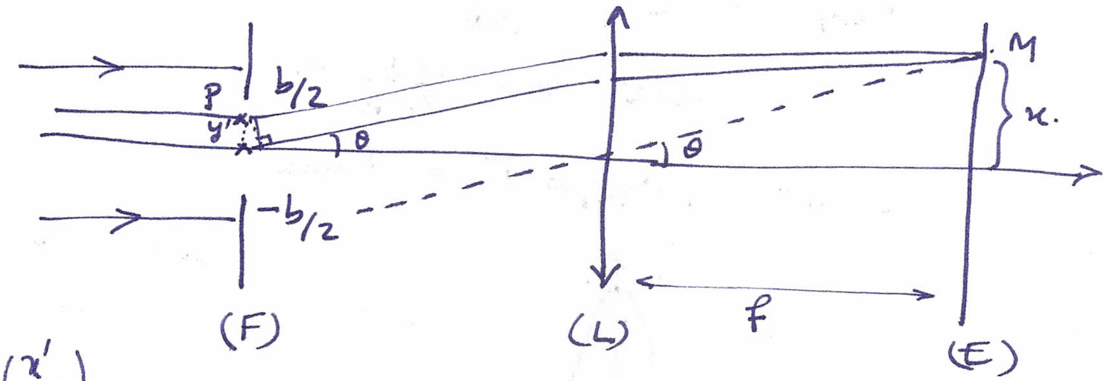
Ex. 10. (2021) Diffraction en lumière blanche

1

1/ $\lambda = 633\,nm$. $b = 120\,\mu m$.

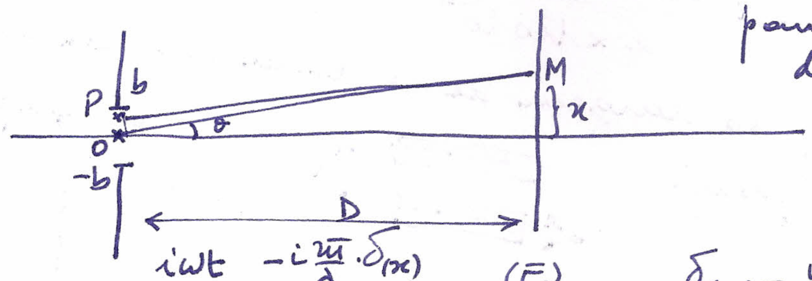


$P\begin{pmatrix} x' \\ y' \\ 0 \end{pmatrix}$ entre P et O (origine de phases en O)

il y a un déphasage $\varphi_{(x)} = \frac{2\pi\delta(x)}{\lambda}$.

où $\delta_{(x)} \simeq y.\theta \simeq y.\frac{x}{f}$.

⟹ [Sans la lentille (condition de Fraunhofer) :



pour un élément de longueur dy' centré sur P on a

$$dE_{P(x,t)} = \frac{E_0}{2b} e^{i\omega t} e^{-i\frac{2\pi}{\lambda}.\delta_{(x)}} dy \qquad \delta_{(x)} = y.\sin\theta \simeq y.\frac{x}{D}.$$

$$E_{(x,t)} = \frac{E_0}{2b} e^{i\omega t} \int_{-b}^{b} e^{-i\frac{2\pi}{\lambda D} y x} dy = \frac{E_0}{2b} e^{i\omega t} \frac{1}{-i\frac{2\pi}{\lambda D}x} \left[e^{-i\frac{2\pi}{\lambda D} y x} \right]_{-b}^{b}$$

$$= \frac{E_0}{2b} e^{i\omega t} \frac{1}{-i\frac{2\pi x}{\lambda D}} \left[e^{-i\frac{2\pi x b}{\lambda D}} - e^{i\frac{2\pi b x}{\lambda D}} \right]$$

$$= \frac{E_0}{2b} e^{i\omega t} \frac{1}{-i\frac{2\pi x}{\lambda D}} \left(-2i \sin\left(\frac{2\pi b x}{\lambda D}\right) \right)$$

$$= \frac{E_0}{\cancel{b}} e^{i\omega t} \frac{1}{\left(\frac{2\pi x b}{\lambda D}\right)} \sin\left(\frac{2\pi x b}{\lambda D}\right)$$

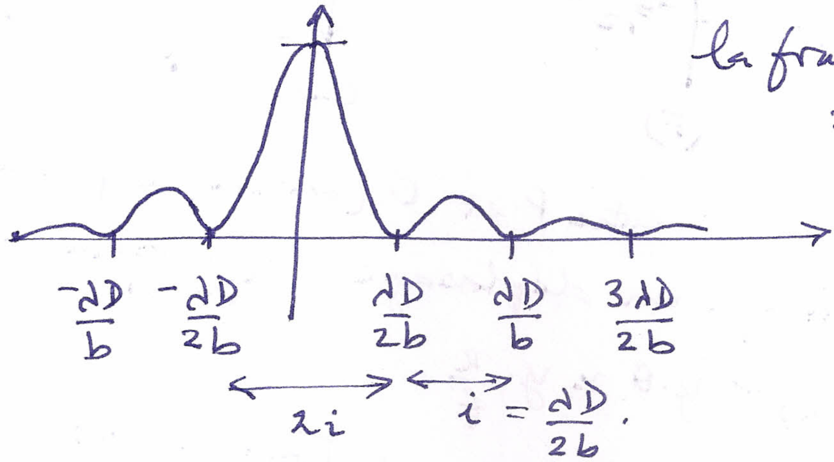
$$\Rightarrow \boxed{I_{(x)} = I_0 \ \mathrm{sinc}^2\left(\frac{2\pi x b}{\lambda D}\right)}.\ ]$$

i/ $$I_{(x)} = I_0 \operatorname{sinc}^2\left(\frac{2\pi b x}{\lambda D}\right).$$

$$I(x) = 0 \iff \frac{2\pi b x}{\lambda D} = k.\pi \quad (k \in \mathbb{Z}^*)$$

soit $x = \frac{\lambda}{2}\frac{D}{b}\,k$.

en $x = 0$, $I_{(x)} = I_0 = I_{max}$.

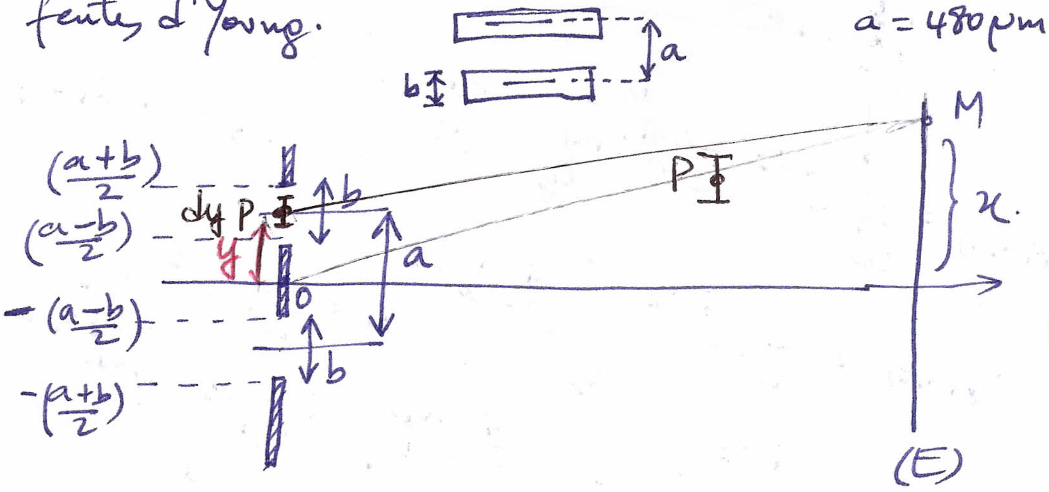


la frange centrale est 2 fois plus large que les autres.

__AN__ : $i = \frac{0{,}633\,10^{-6} \times 1}{2 \times 120\,10^{-6}} = 2.6\,10^{-3}\ m$.

la largeur de la tache centrale est $5{,}2\,10^{-3}\ m$.

ii/ fentes d'Young. $a = 480\,\mu m$.



__origine des phases en O__

$$y = y_p \in \left[-\left(\frac{a+b}{2}\right), -\left(\frac{a-b}{2}\right)\right] \cup \left[\frac{a-b}{2}, \frac{a+b}{2}\right] = \mathcal{E}$$

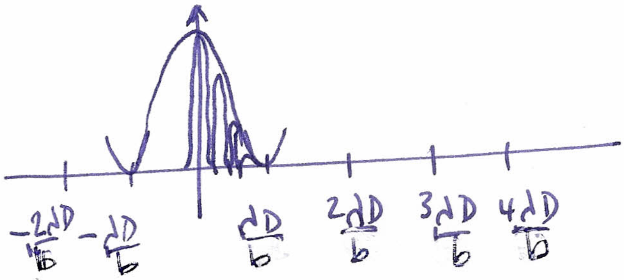
$$\int dE_{p(x,t)} = \frac{E_0}{S} e^{i\omega t}\left[\int_{\mathcal{E}} e^{-i\frac{2\pi}{\lambda}\delta}\,dy\right]$$

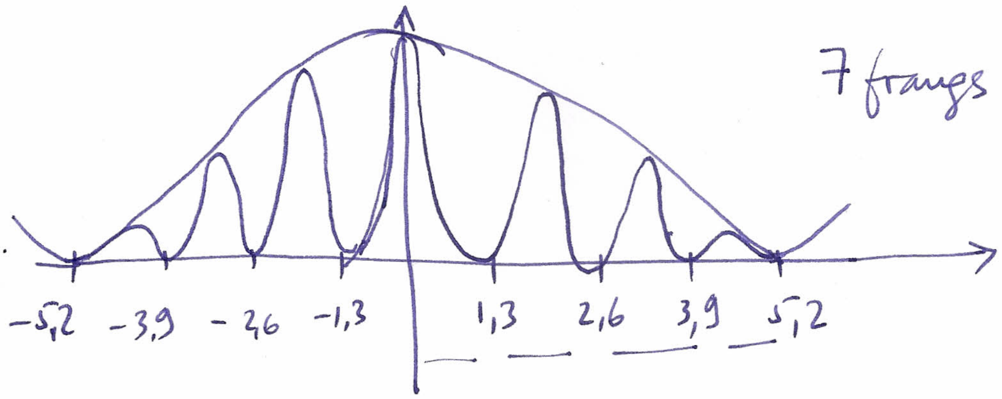
$$E_{(M,t)} = \frac{E_0}{S} e^{i\omega t}\left[\frac{\sin\left(\frac{\pi b x}{\lambda D}\right)}{\frac{\pi b x}{\lambda D}}\right] \times \cos\left(\frac{\pi a x}{\lambda D}\right).$$

$$\Rightarrow I_{(M)} = I_0 \, \frac{\sin^2\left(\frac{\pi b x}{\lambda D}\right)}{\left(\frac{\pi b x}{\lambda D}\right)^2} \times \cos^2\left(\frac{\pi a x}{\lambda D}\right)$$

$$\sin\left(\frac{\pi b x}{\lambda D}\right) = 0 \iff \frac{\pi b x}{\lambda D} = k\pi \Rightarrow x_k = k \cdot \frac{\lambda D}{b} = k \cdot 5{,}2 \cdot 10^{-3}$$

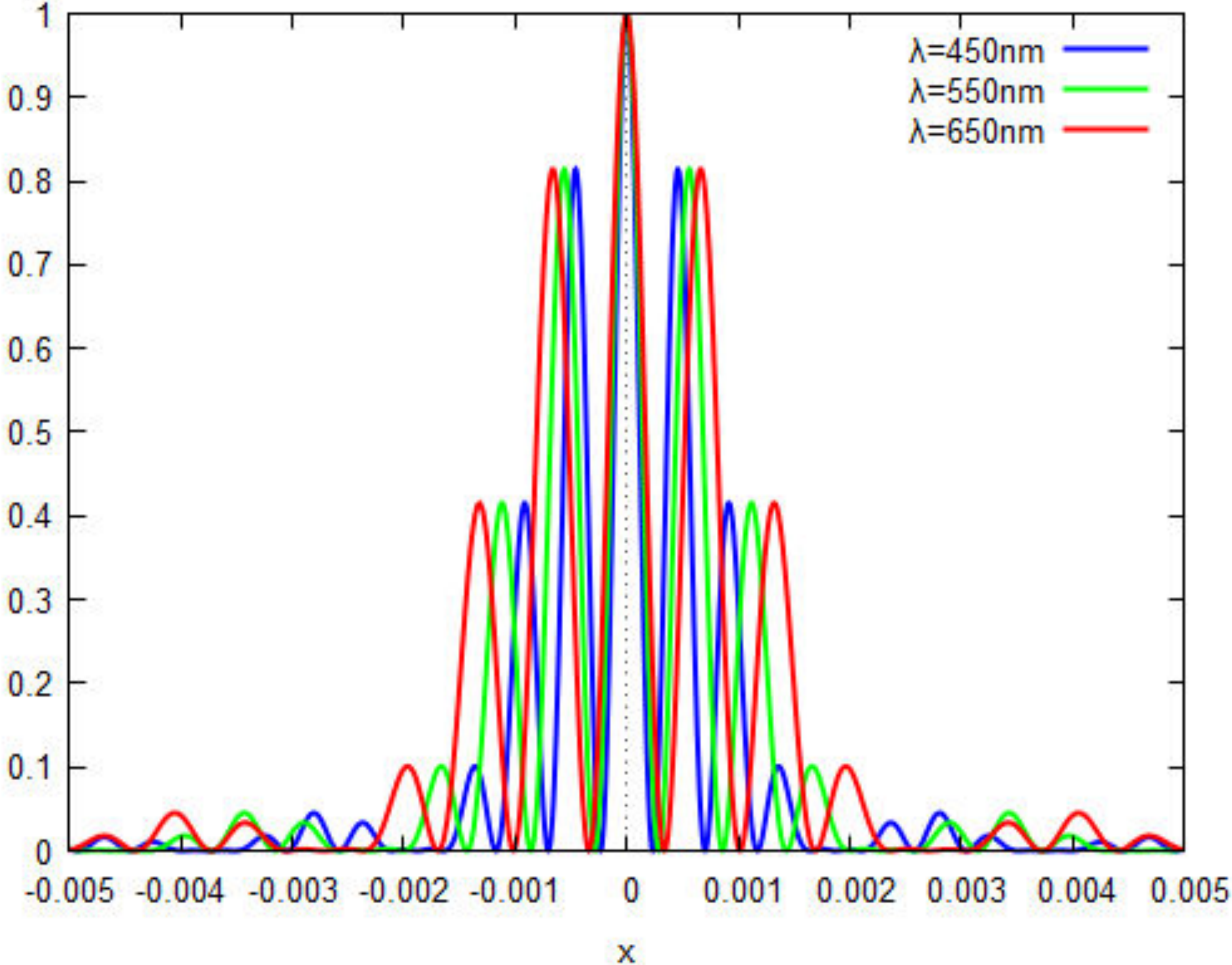
$$\cos\left(\frac{\pi a x}{\lambda D}\right) = 0 \iff \frac{\pi a x}{\lambda D} = p \cdot \pi \Rightarrow x_p = p \cdot \frac{\lambda D}{a} = p \times 1{,}3 \cdot 10^{-3}$$



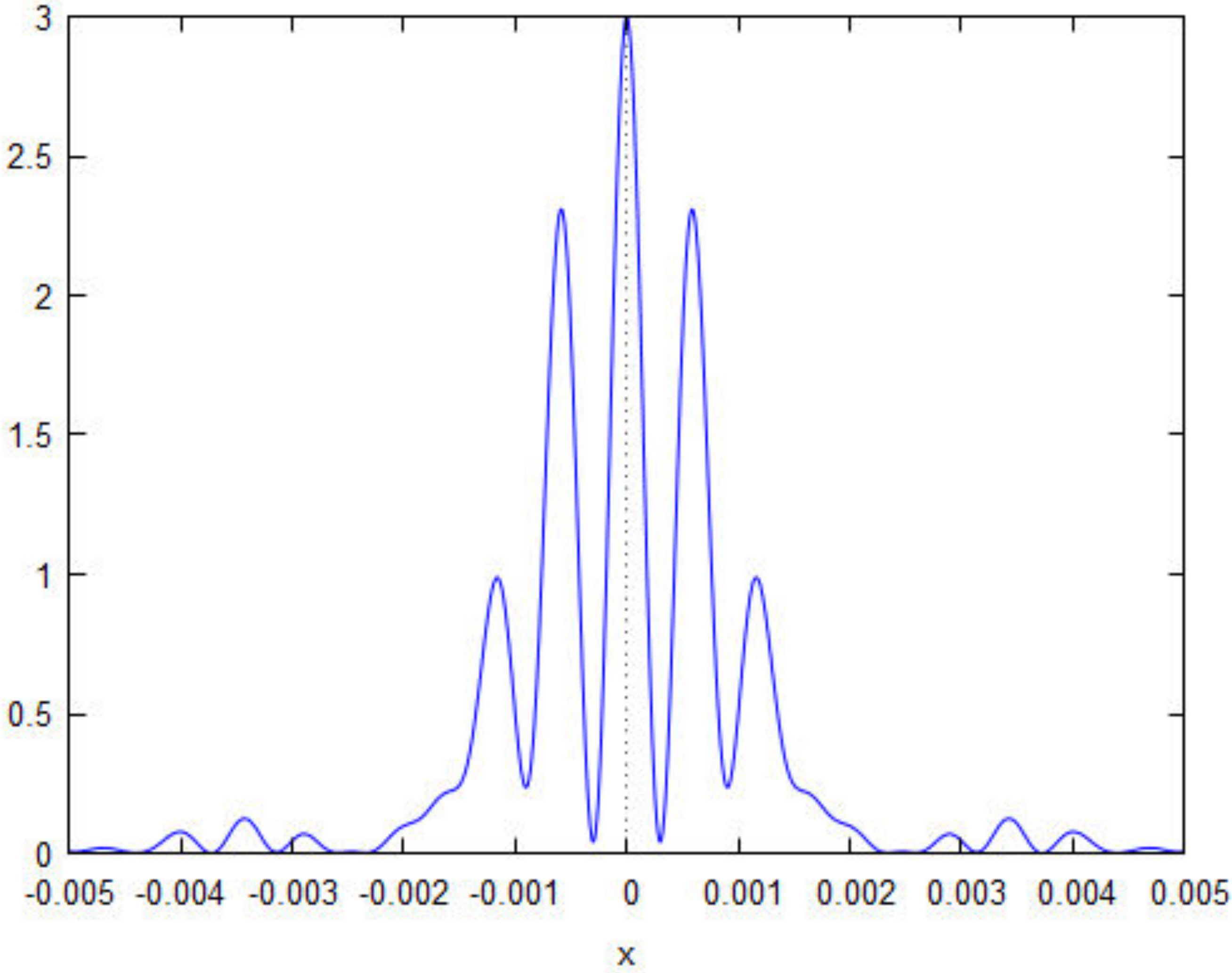


7 franges brillants.

3/ En lumière blanche : voir figure.



λ=450nm
λ=550nm
λ=650nm
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.005
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
0.005
x


3
2.5
2
1.5
1
0.5
0
-0.005
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
0.003
0.004
0.005
x