Advanced biomedical signal and image processing

Master: Plasturgy & Biomedical Engineering

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Faculté de Science Meknes

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Section 1 :

Introduction to Digital Signal and Image Processing

Chapter 2:

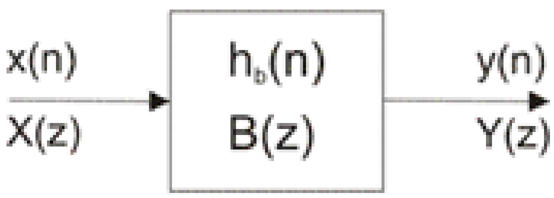
Chapter 2: Fourier analysis of continuous-time signals

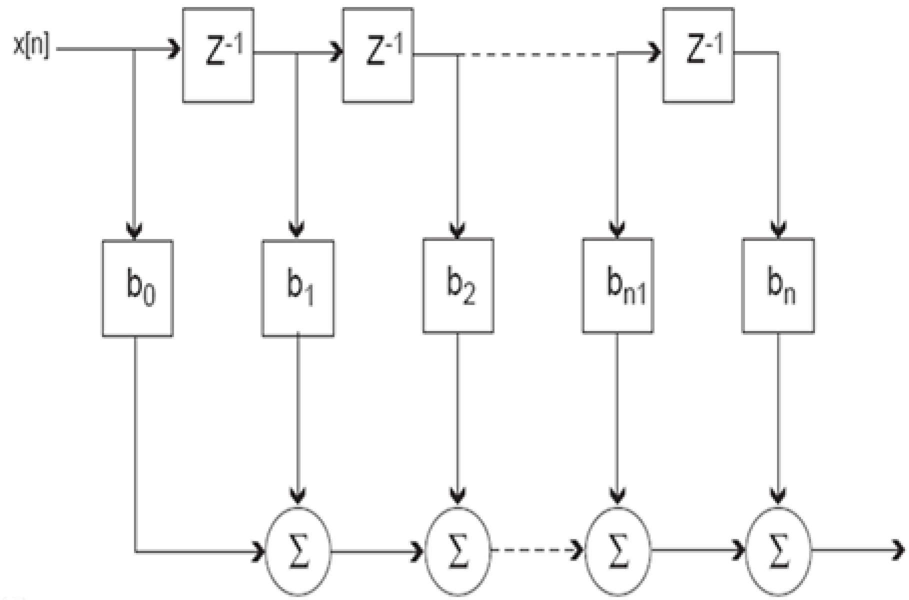
Filters

Other signal processing techniques

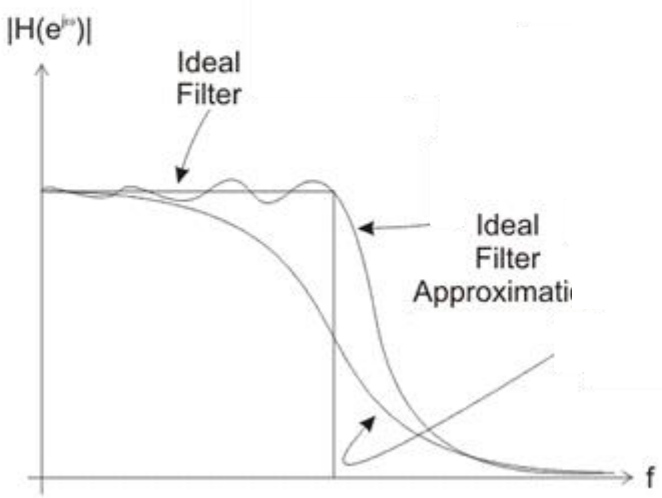
Wavelet transformation

Digital Filters – FIR filters





$$y[n] = \sum_{k=0}^{N} b_k \cdot x[n-k]$$



Ideal low-pass filter approximation

- finite impulse response, no recursion (output does not depend on the input)
- described by multiplication coefficients
- less sufficient (need higher order)

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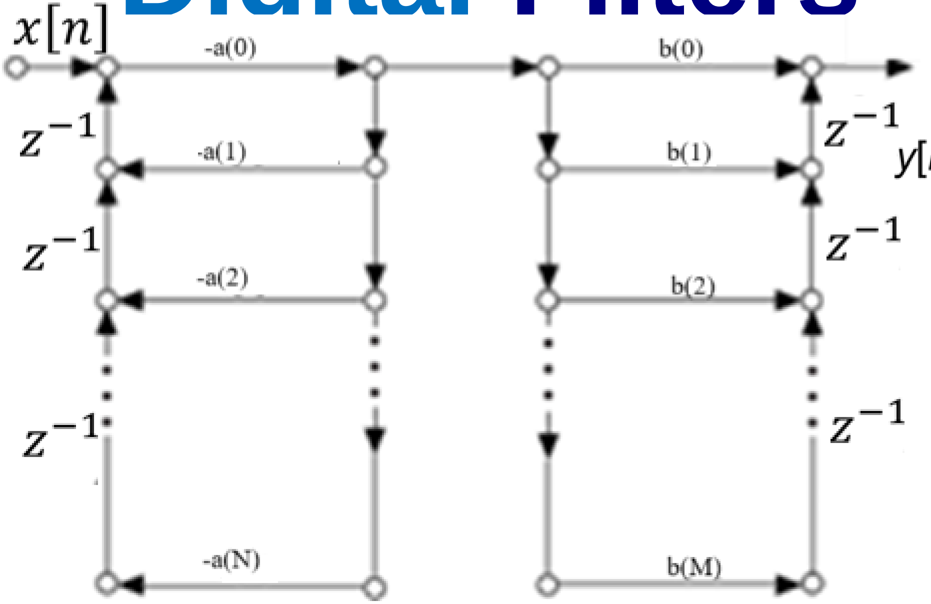
Digital Filters – FIR filters

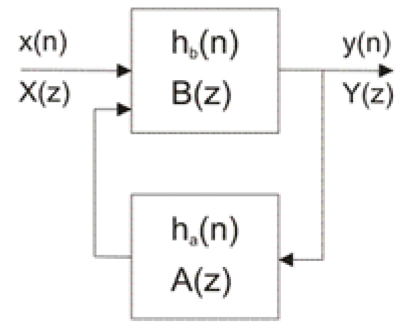
Finite Impulse Response (FIR) filters are widely used in biosignal processing due to several key advantages

- Linear Phase Response that is critical for medical applications like ECG (Electrocardiogram), EEG (Electroencephalogram), and EMG (Electromyogram).
- FIR filters are integrally stable. This stability is crucial for long-term signal monitoring and real-time applications.
- Less oscillations or artifacts in biosignals, which reduce misinterpretation in medical diagnoses.
- Precise control over the frequency response, making them suitable for applications such as: noise removal
- Easy to implement in real-time digital signal processing (DSP) applications, including portable and embedded biomedical devices.
- Suitability for adaptive filtering

Digital Filters – IIR filters



$$y[n] = \sum_{k=0}^{N} b_k \cdot x[n-k] + \sum_{m=1}^{M} -a_m \cdot y[n-m]$$



- infinite impulse response, truncated at a certain precision
- use previously calculated values from the output (recursion)
- described by recursion coefficients
- more efficient
- can be unstable

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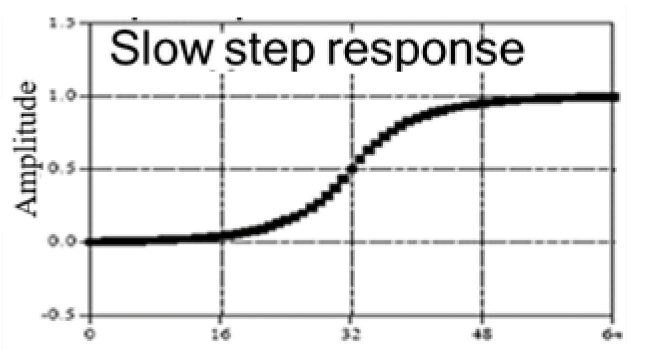
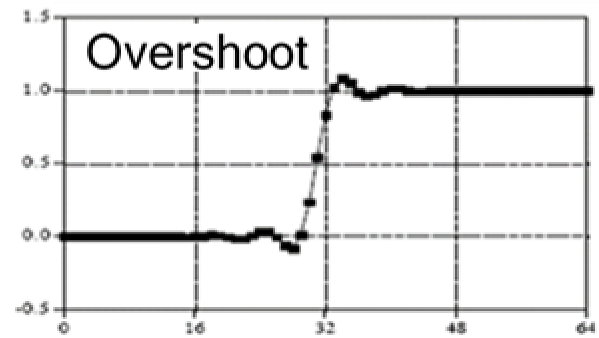
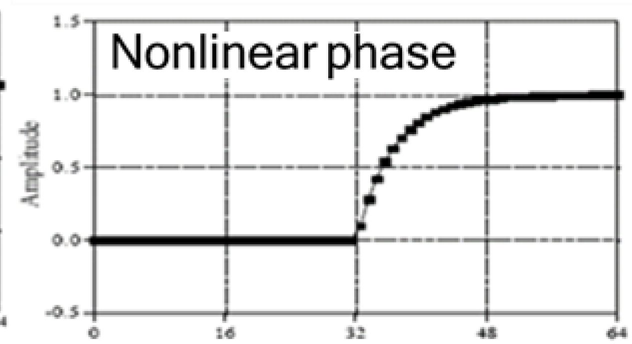
Digital Filter Characteristics

Performance in Time Domain

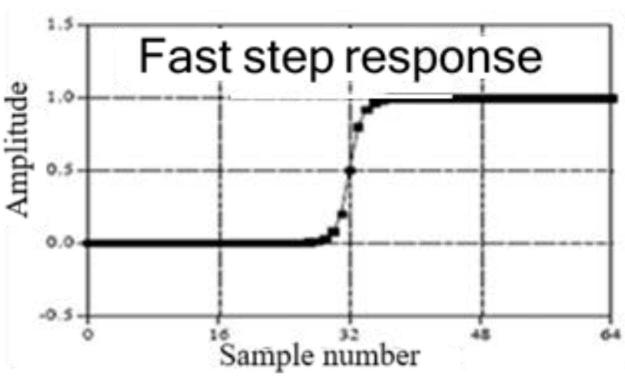
Poor

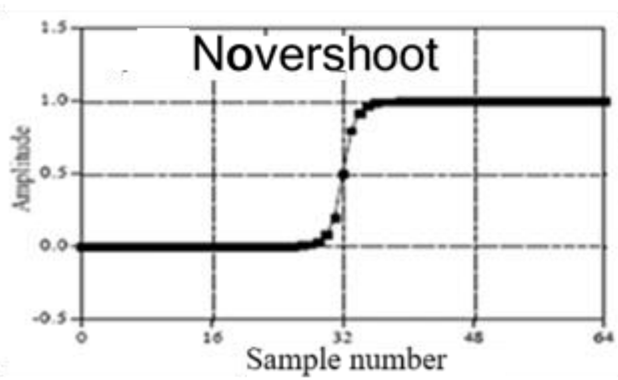


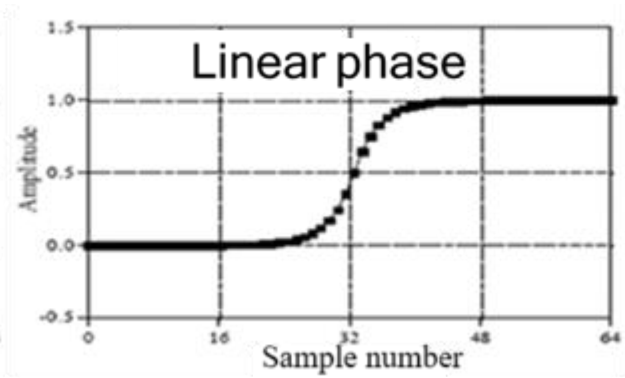




Good







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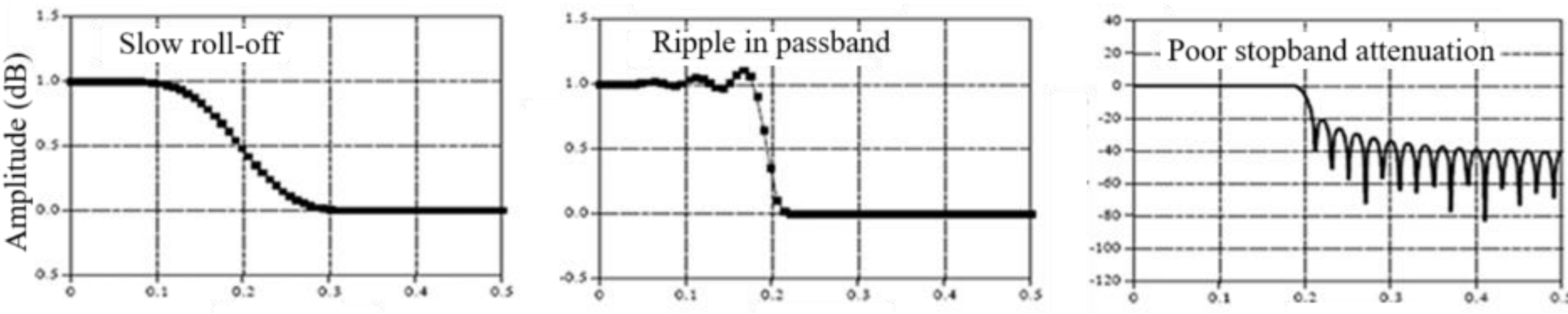
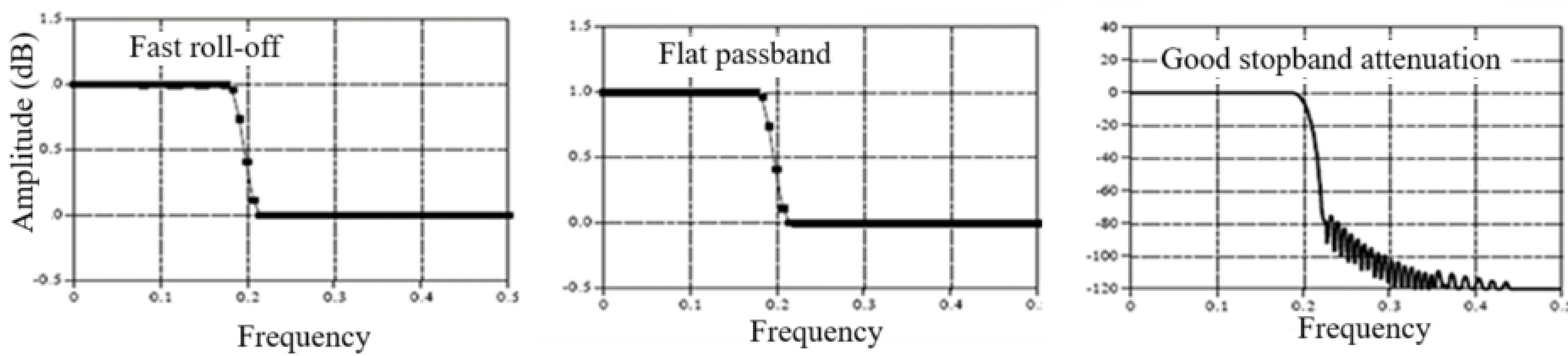
Digital Filter Characteristics

Performance in Frequency Domain

Poor



Good



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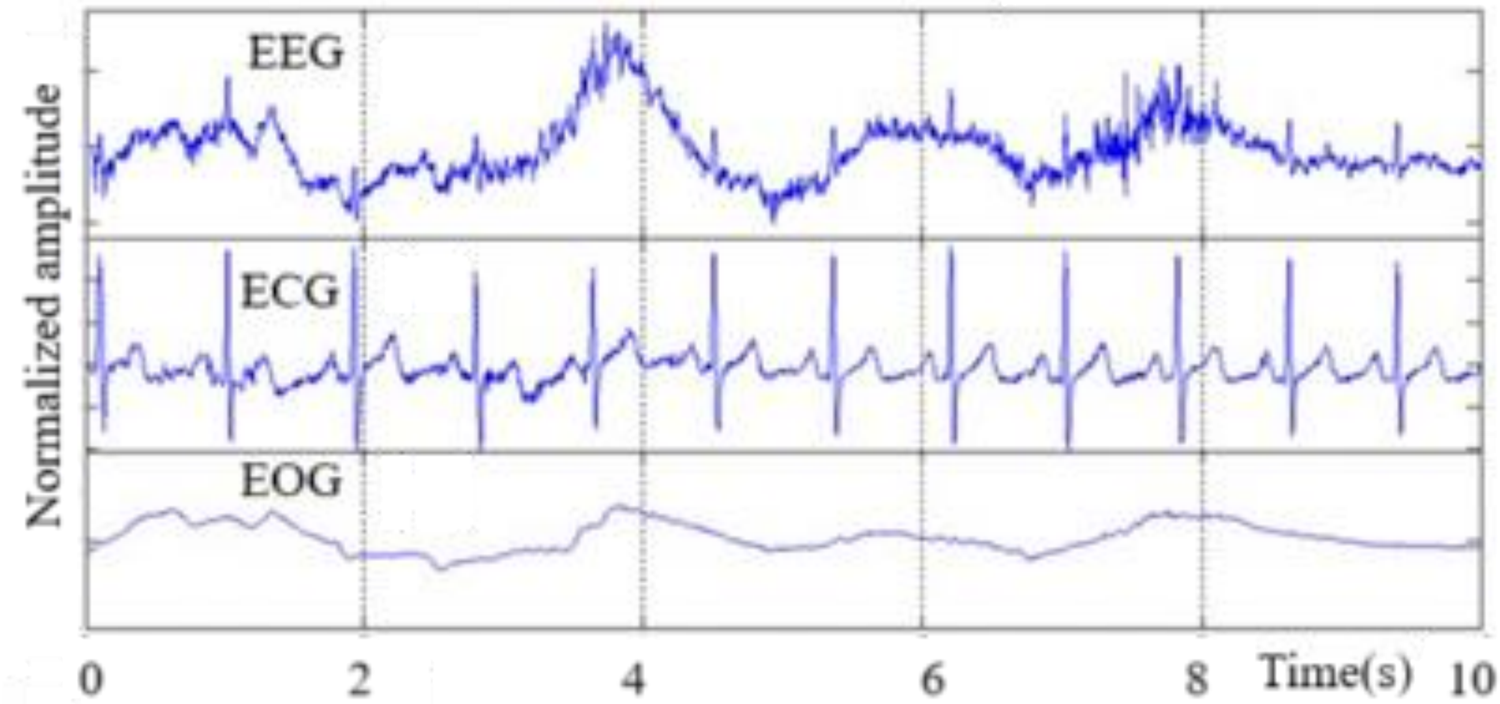
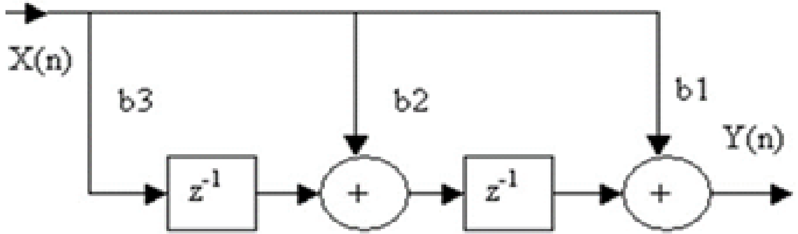
Digital Filter Characteristics-Examples

Examples of three biological signals with their frequency spectrum



Digital FIR Filters
example: Finite Impulse Response Notch Filter

System function:



Filter Coefficients:

$b_1=1$

$b_2=-2\cdot\cos(\omega_0)$

$b_3=1$

Transfer function:

$$H(z)=\frac{Y(z)}{X(z)}=\frac{(z-z_1)\cdot(z-z_2)}{z^2}$$

$$=\frac{z^2-z\cdot z_2-z_1\cdot z+z_1\cdot z_2}{z^2}$$

Scaling coefficient

$$G=\frac{1}{(2-2\cos(\omega_0))}$$

$$=1-z^{-1}\cdot(z_2+z_1)+z^{-2}\cdot z_1\cdot z_2$$

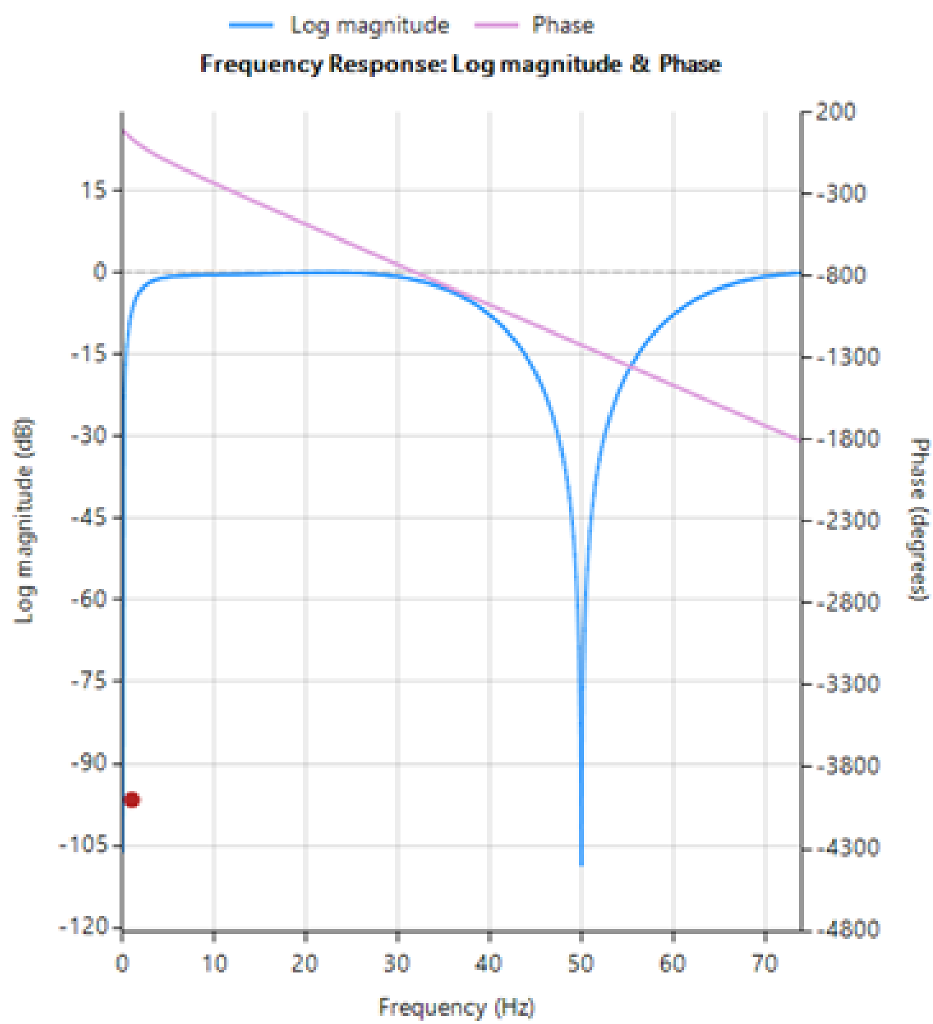
$$z_1=\cos(\omega_0)+j\cdot\sin(\omega_0)$$

$$z_2=\cos(\omega_0)-j\cdot\sin(\omega_0$$

$$H(z)=1+z^{-1}(-2(\cos(\omega_0))+z^{-2}$$

$$=b_1+b_2z^{-1}+b_3z^{-2}$$

Digital FIR Filters
60 Hz notch filter characteristics



Design Digital FIR Filters
60 Hz notch filter example

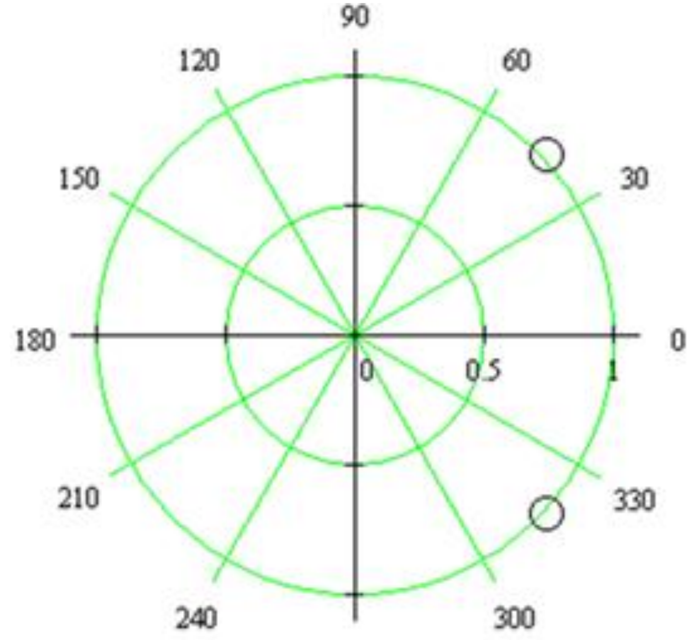
Frequencies that define complex zeros:

f_0=60Hz - power supply frequency

f_s=500Hz - sampling rate

$$\omega_0 = 2 \cdot \pi \cdot \frac{f_0}{f_s}$$

we get $\omega_0 = 0.754$



Positions of complex zeros:

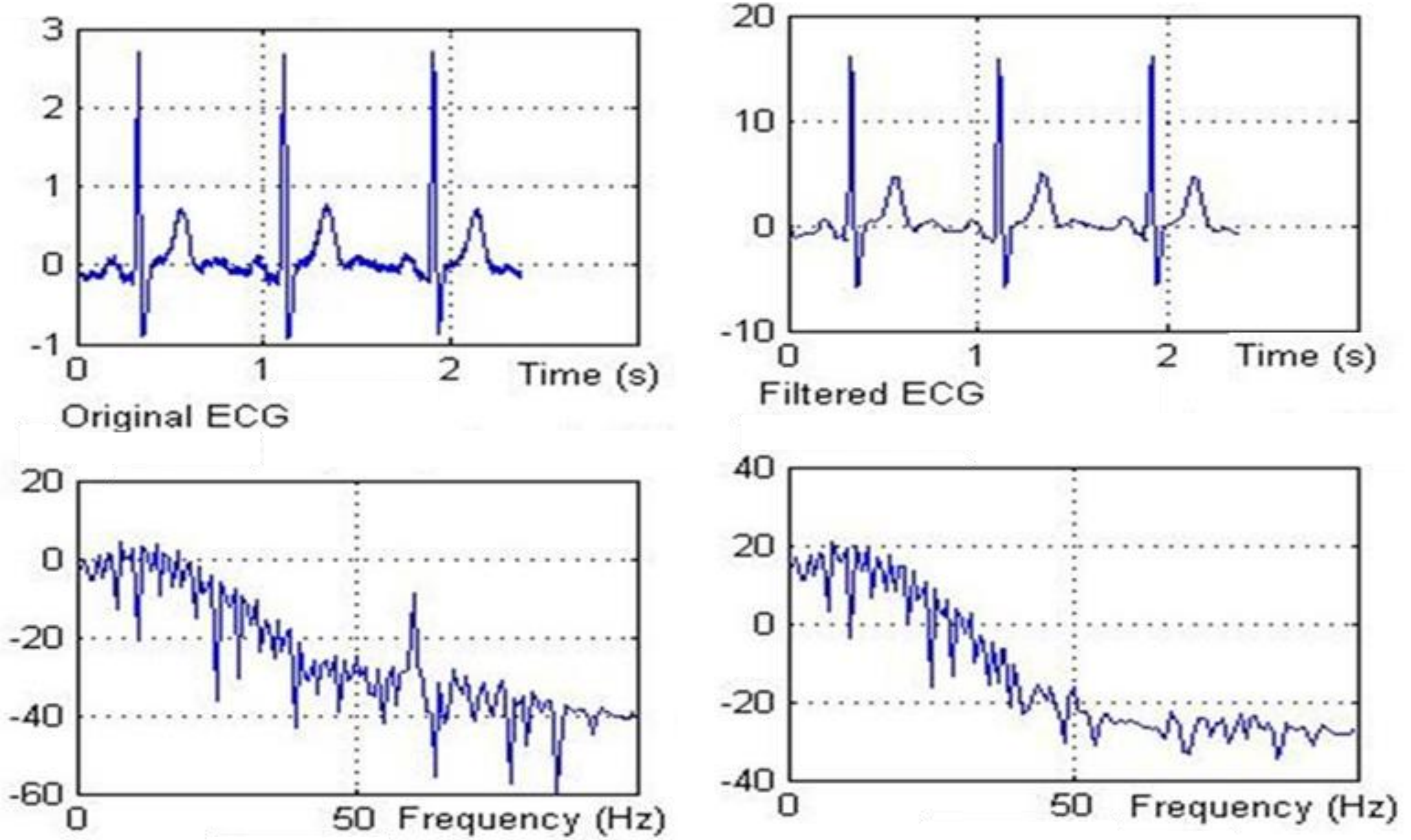
$$z_1 = \cos(\omega_0) + j \cdot \sin(\omega_0)$$

$$z_2 = \cos(\omega_0) - j \cdot \sin(\omega_0)$$

$\arg[Z_1]. arg[Z_2]$

Design digital FIR Filters
60 Hz notch filter - implementation in Matlab

60Hz notch applied to ECG signal

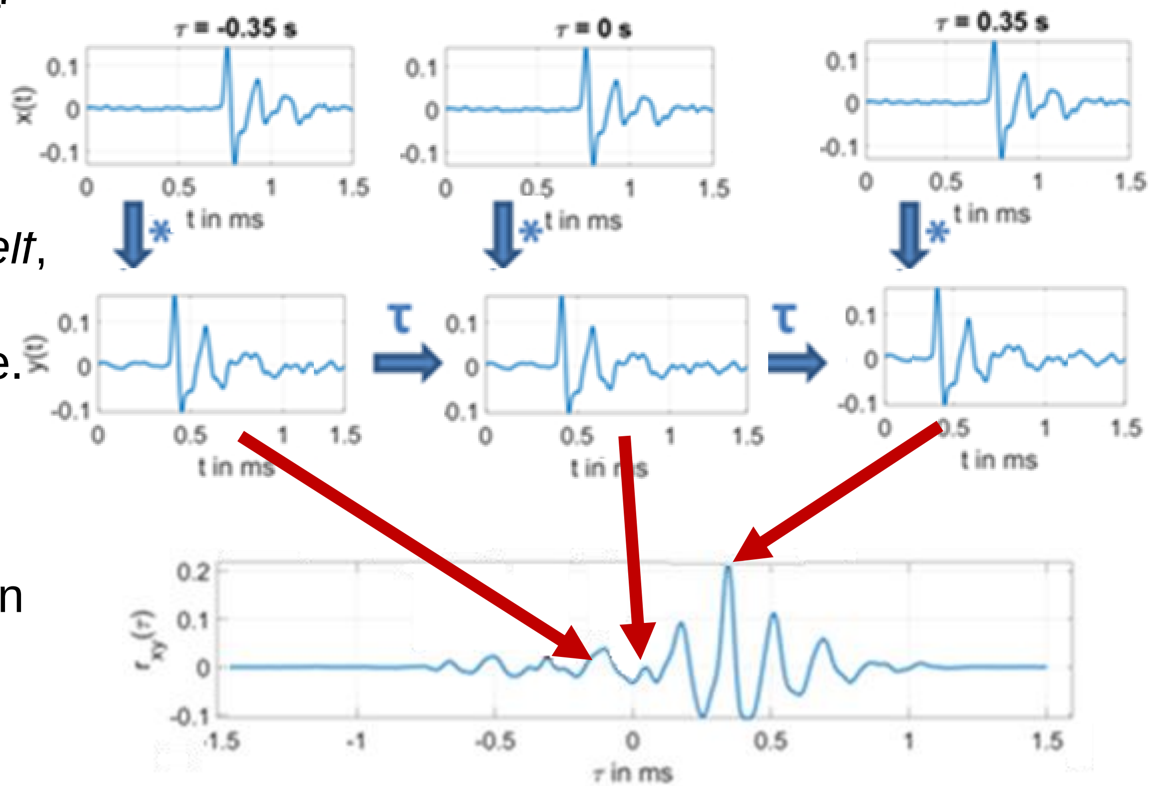


Other Signal processing techniques correlation

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)\, x^{\star}(t-\tau)\, dt$$

$$R_{xx}[m] = \sum_{n=-\infty}^{\infty} x[n]\, x^{\star}[n-m]$$

- mathematical operation that is very similar to convolution
- uses two signals to produce a third signal. This third signal is called the **cross-correlation** of the two input signals (i.e.finds similar signals in a signal)
- if a signal is correlated with *itself*, the resulting signal is instead called the **auto-correlation** (i.e. finds periodic parts of a signal)
- Correlation is the *optimal* technique for detecting a known waveform in random noise.



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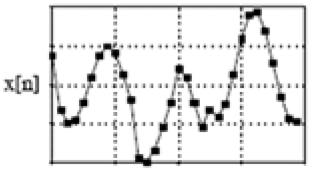
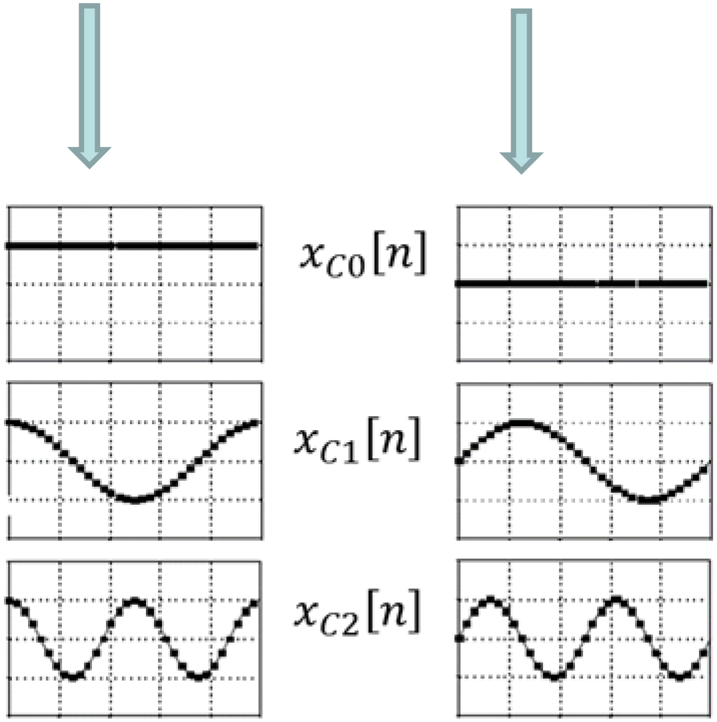
Other signal processing techniques discrete Fourier transform (DFT)

- Decomposition of a signal into



sine and cosine waves



Other signal processing techniques discrete Fourier transform (DFT)

Fourier Analysis of Discrete Time Signals mathematical concept

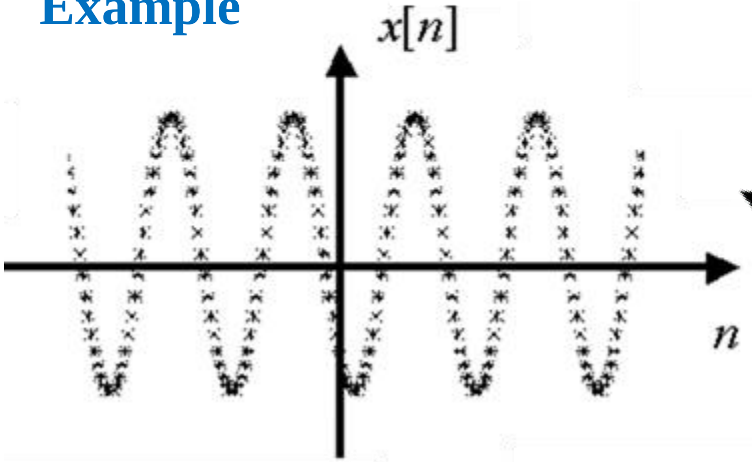
For a discrete time sequence we define two classes of Fourier Transforms:

- DTFT (Discrete Time FT) for sequences having **infinite** duration,
- DFT (Discrete FT) for sequences having **finite** duration.

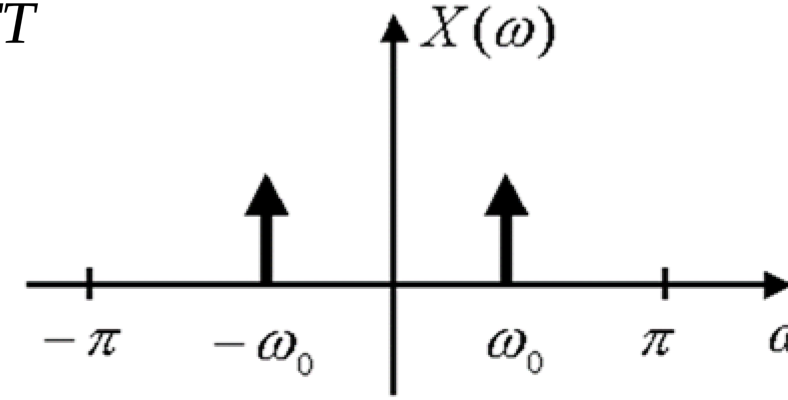
Other signal processing techniques discrete Fourier transform (DFT)

Fourier Analysis of Discrete Time Signals mathematical concept

Example



DTFT



$$x[n] = A\cos(\omega_0 n + \alpha)$$

$$X(\omega) = A\pi e^{j\alpha}\delta(\omega - \omega_0) + A\pi e^{-j\alpha}\delta(\omega + \omega_0)$$

Other signal processing techniques discrete Fourier transform (DFT)

Fourier Analysis of Discrete Time Signals mathematical concept

Discrete Fourier Transform (DFT)

Consider the finite discrete sequence

$$x = [x(0), x(1), \ldots, x(N-1)]$$

its Discrete Fourier Transform (DFT) is a finite sequence

$$X = DFT(x) = [X(0), X(1), \ldots, X(N-1)]$$

where

$$X(k) = \sum_{n=0}^{N-1} x(n) w_N^{kn} \qquad w_N = e^{-j2\pi/N}$$

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Other signal processing techniques discrete Fourier transform (DFT)

Fourier Analysis of Discrete Time Signals mathematical concept

Inverse Discrete Fourier Transform (IDFT)

Consider the frequency series

$$X = [X(0), X(1), \ldots, X(N-1)]$$

its Inverse Discrete Fourier Transform (IDFT) is a finite sequence

$$x = IDFT(X) = [x(0), x(1), \ldots, x(N-1)]$$

where

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) w_N^{-kn}$$

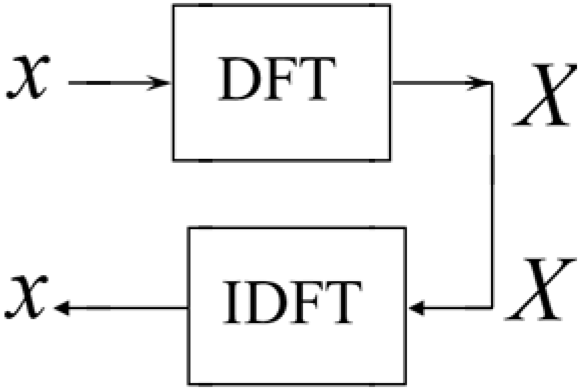
$$w_N = e^{-j2\pi/N}$$

Other signal processing techniques discrete Fourier transform (DFT)

Fourier Analysis of Discrete Time Signals mathematical concept

Note that:

The DFT and the IDFT form a transform pair.

back to the same signal 

The DFT is a numerical algorithm, and it can be computed by a digital computer.

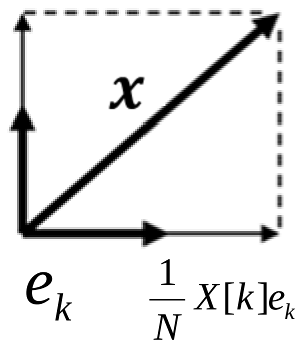
Other signal processing techniques discrete Fourier transform (DFT)

DFT as vector operation

Let consider a discreet sequence as a vector $x = \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$,

Let consider as well the vector base: $e_k = \begin{bmatrix} 1 \\ w_N^{-k} \\ \vdots \\ w_N^{-k(N-1)} \end{bmatrix}$,

$$x = \frac{1}{N}\left(X[0]e_0 + X[1]e_1 + \ldots + X[N-1]e_{N-1}\right)$$



DFT of this discreet sequence is also as vector

$$X = DFT\{x\} = \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} \qquad X[k] = e_k^{*T} x$$

Other signal processing techniques discrete Fourier transform (DFT)

DFT as vector operation

$$X = DFT\{x\} = \begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[N-1] \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & w_N & \cdots & w_N^{N-1} \\ \vdots & & \ddots & \\ 1 & w_N^{N-1} & & w_N^{(N-1)(N-1)} \end{bmatrix}}_{W_N} \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[N-1] \end{bmatrix}$$

$$\Rightarrow \quad X = W_N x$$

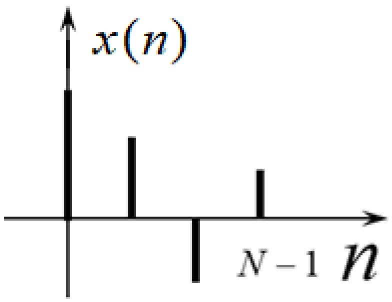
$$x = \underbrace{\frac{1}{N} W_N^{*T}}_{W_N^{-1}} X$$

Other signal processing techniques discrete Fourier transform (DFT)

Periodicity

IDFT expression, shows that the sequence *x(n)* is interpreted as one period sequence $x_p(n)$

original sequence



$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) w_N^{-kn}$$

We can multiply by w_N^{-kN} since $w_N^{-kN} = e^{-j2\pi kN/N}$

$$= e^{-j2\pi}$$

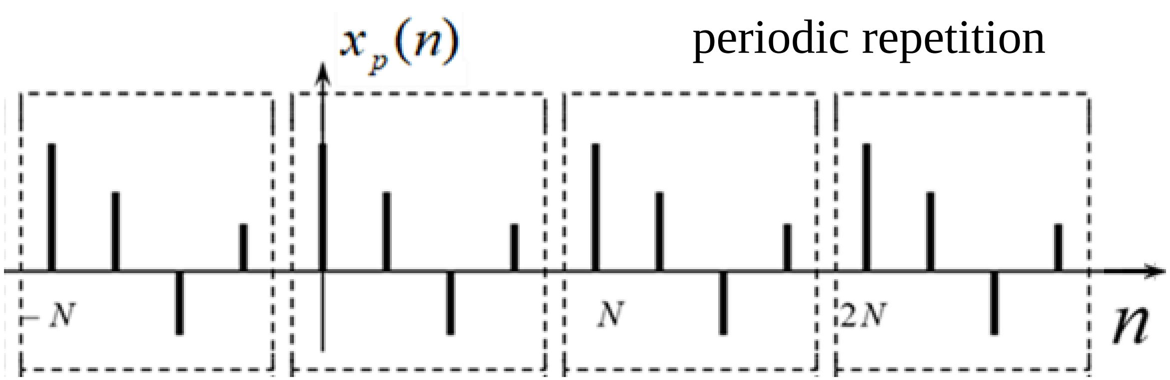
$$=1$$

$$x_p(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k) w_N^{-kn} w_N^{-kN}$$

$$= \frac{1}{N}\sum_{k=0}^{N-1} X(k) w_N^{-k(n+N)}$$

$$= x_p(n+N)$$

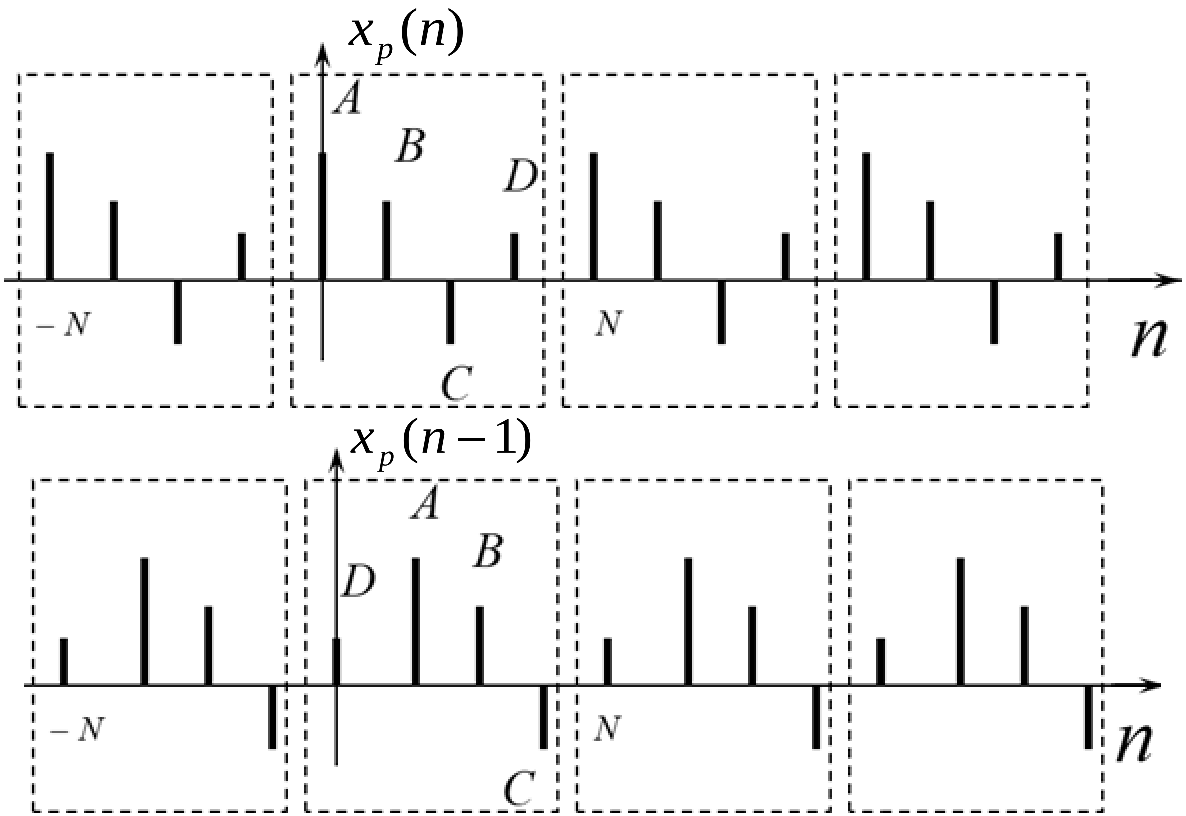
periodic repetition



Other signal processing techniques discrete Fourier transform (DFT)

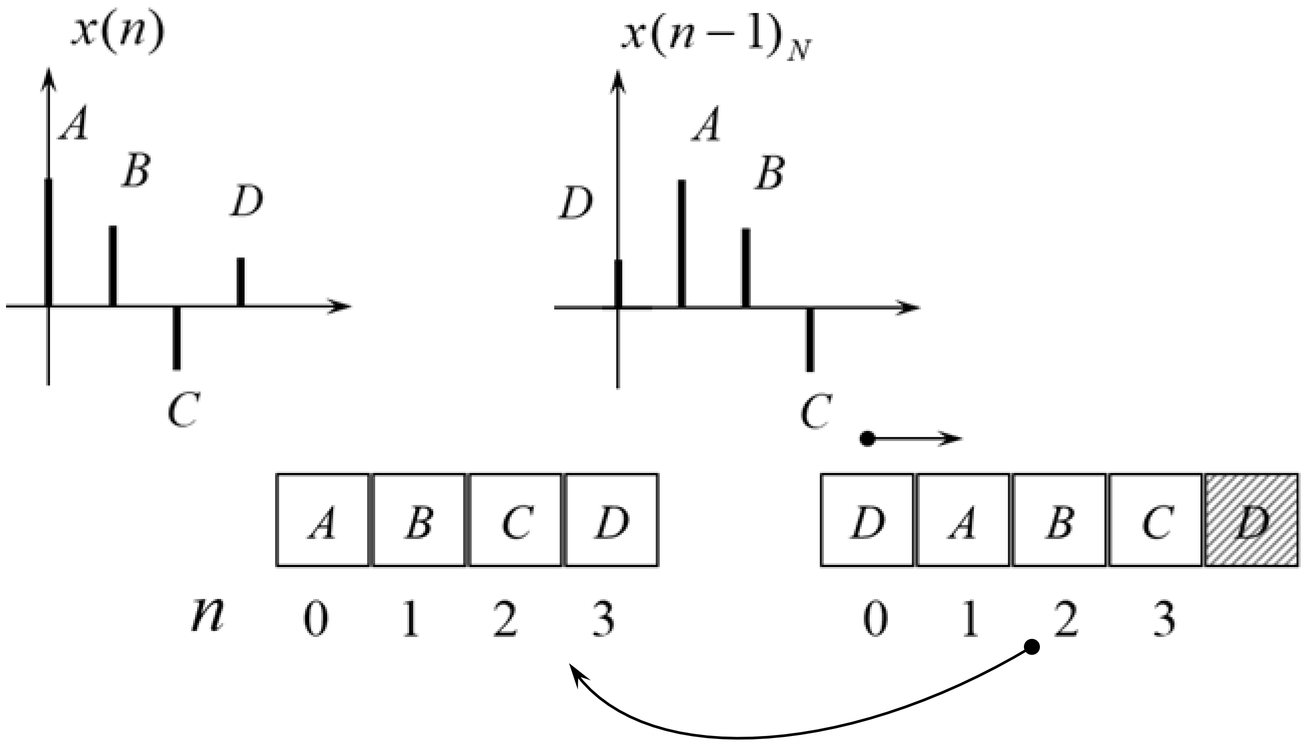
Time shift of the sequence.

For example see what we mean with $x(n-1)$. Start with the periodic extension $x_p(n)$



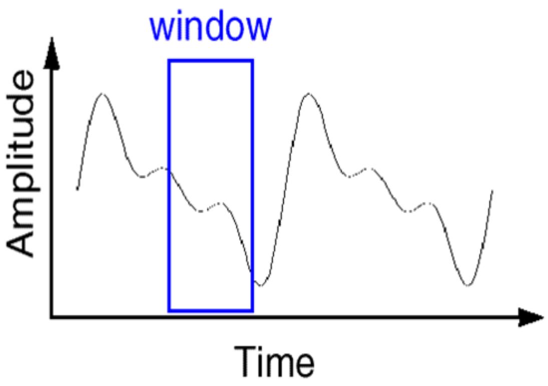
Other signal processing techniques discrete Fourier transform (DFT)

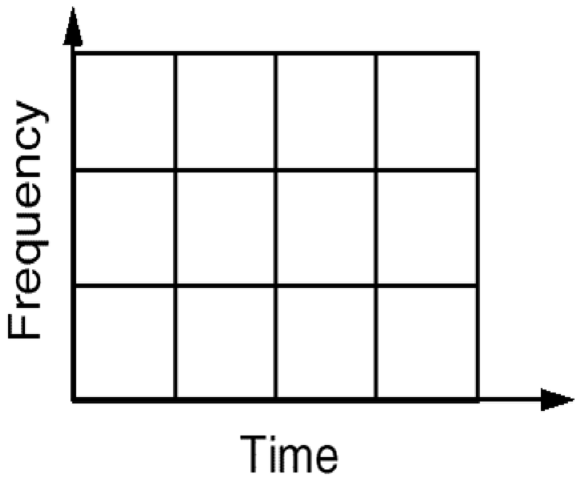
Circular shift of the sequence at time period



Short time Fourier Analysis

- In order to analyze small section of a signal, Denis Gabor (1946), developed a technique, based on the FT and using *windowing* : STFT (Short time Fourier transform)
- A compromise between time-based and frequency-based views of a signal.
- both time and frequency are represented in limited precision.
- The precision is determined by the size of the window.
- Once you choose a particular size for the time window - it will be the same for all frequencies.
- Many signals require a more flexible approach
- We can vary the window size to determine more accurately either time or frequency.



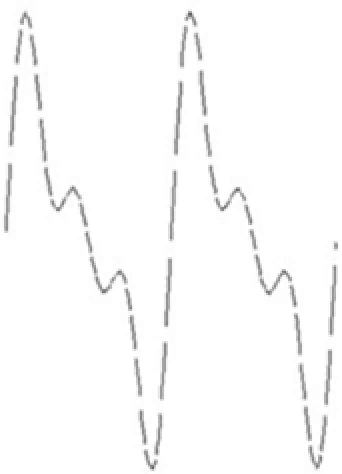


The Continuous Wavelet Transform (CWT)

- The Fourier transform for a continuous data or a signal given by the function $f(t)$ is:

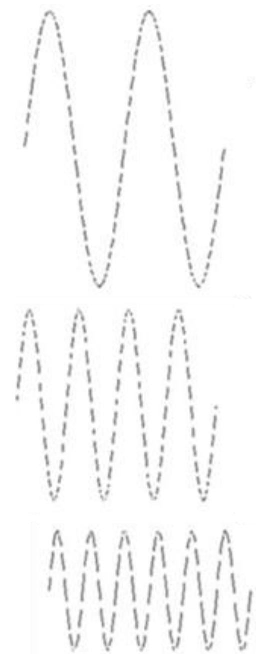
$$F(\omega) = \int_{-\infty}^{+\infty} f(t) e^{-j\omega} dt$$

- is composed of multiple sinusoids with different frequencies and amplitudes



Signal $f(t)$

Fourier transform $F(\omega)$



Signal $F(\omega)$

Wavelet Transformation- Equations

- Wavelet Transform $$\gamma(s,\tau) = \int f(t)\psi^{*}_{S,T}(t)dt$$
- Inverse Wavelet Transform $$f(t) = \iint \gamma(s,\tau)\psi_{S,T}(t)d\tau\, ds$$
- Mother wavelet $$\psi_{S,T}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

Wavelet Transformation- Scaling

Proprieties

normalization

shift in time

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

change in scale: big s means long wavelength

wavelet with scale, s and time, τ

Mother wavelet

Wavelet transform

Proprieties

time-series

$$\gamma(s,\tau) = \int f(t)\psi^{*}_{s,\tau}(t)dt$$

conjugate from now on, assuming that we're using real wavelets

coefficient of wavelet with scale, s and time, τ

complex conjugate of wavelet with scale, s and time, τ

Inverse Wavelet Transform

$$f(t) = \iint \gamma(s,\tau)\psi_{S,T}(t)\,d\tau\,ds$$

time-series

coefficients of wavelets

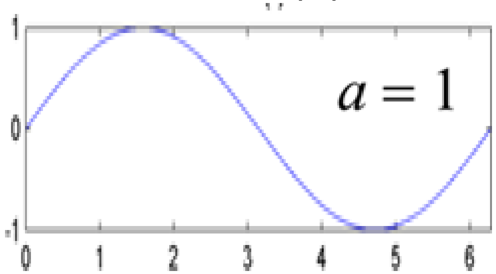
wavelet with scale, s and time, τ

Watch this video

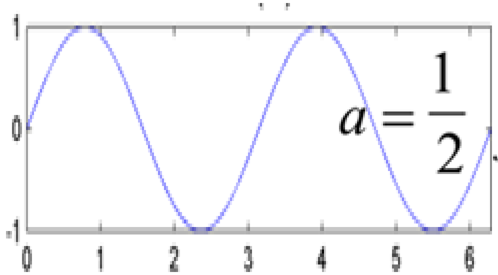
Wavelet Transformation- Scaling

- Wavelet analysis produces a time-scale view of the signal.
- *Scaling* means stretching or compressing of the signal.

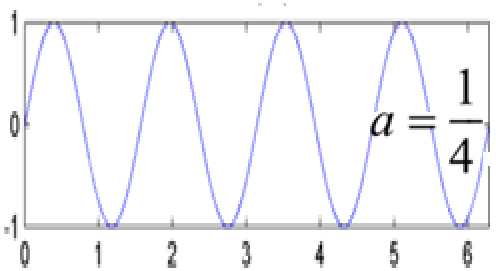
Scale factor (a) for sine waves:



$$f(t) = \sin(t)$$



$$f(t) = \sin(2t)$$

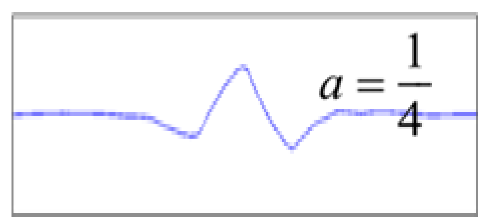


$$f(t) = \sin(4t)$$

Scale factor (a) with wavelets:

$$f(t) = \Psi(t)$$

$$f(t) = \Psi(2t)$$



$$f(t) = \Psi(4t)$$

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