Advanced biomedical signal and image processing

Master: Plasturgy & Biomedical Engineering

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Faculté de Science Meknes

Professor Omar ELOUTASSI

Section 1 :

Introduction to Digital Signal and Image Processing

Chapter 3:

Image Filtering, enhancement, and restoration

Image Filtering

Gaussian Filter

widely used filter in image processing for reducing noise and smoothing images:

value of the Gaussian filter at coordinates (x,y)

$$G(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

standard deviation of the Gaussian distribution, which controls the amount of smoothing.

$2\pi\sigma^2$ normalizes the filter so that the sum of all coefficients equals 1.

Properties

Smoothing: The Gaussian filter smooths an image by averaging pixel values with their neighbors in a weighted manner, where pixels closer to the center have a higher weight.

Isotropic: The filter is isotropic, meaning it has the same effect in all directions.

Controlled Blurring: The degree of blurring can be controlled by adjusting σ. A larger σ results in more smoothing.

Example

Applying a Gaussian Filter:

Consider a 5x5 kernel for a Gaussian filter with $\sigma = 1$

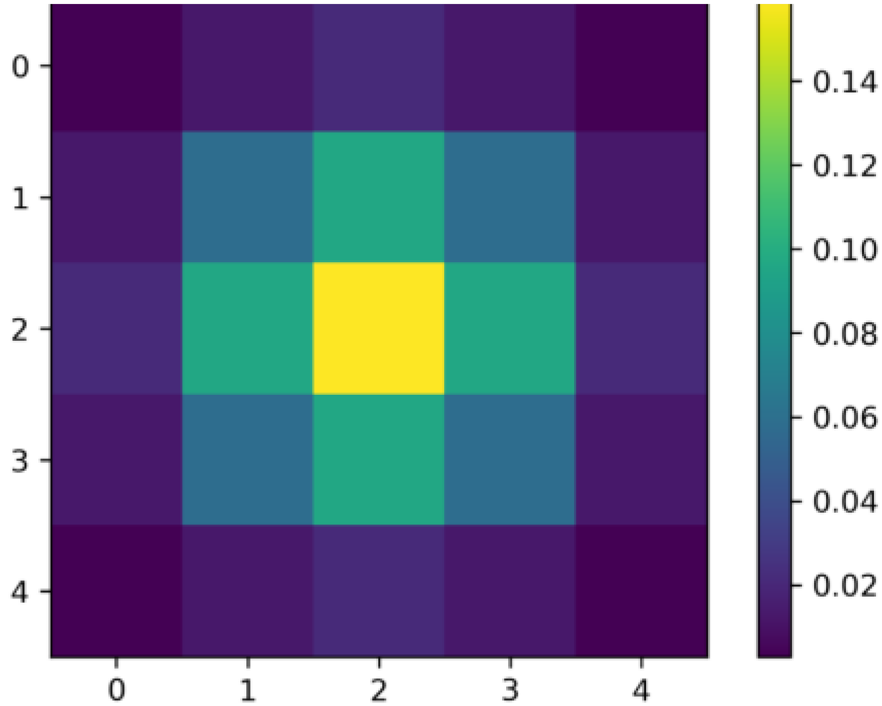
$$\begin{bmatrix} 0.0613 & 0.1247 & 0.0613 \\ 0.1247 & 0.2486 & 0.1247 \\ 0.0613 & 0.1247 & 0.0613 \end{bmatrix}$$

When this kernel is convolved with an image, each pixel in the output image is computed as the weighted sum of the neighboring pixels based on the kernel values. This results in a smoother image with reduced noise.

To visualize the effect of a Gaussian filter, consider a noisy image. After applying the filter, the image appears smoother, and the noise is less pronounced.

Example

$$h(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



2-d visualization of a Gaussian function.

Image enhancement

Contrast stretching

is a technique used to enhance the contrast of an image by expanding the range of intensity values as:

the original pixel value at coordinates (*x*,*y*)

the minimum pixel value in original value image

$$I_{enhanced}(x, y) = \frac{I(x,y) - I_{min}}{I_{max} - I_{min}} \times (L - 1)$$

enhanced pixel value at coordinates (*x*,*y*)

The maximum pixel values in the original image

the number of intensity levels (e.g., 256 for an 8-bit image)

Steps for contrast stretching

Find minimum and maximum:

Determine I_{min} and I_{max} from the original image

Apply transformation: Use the contrast stretching formula to compute the new pixel values

Resulting image: The resulting image will have its pixel values spread across the full range, enhancing the visual contrast

Example

Let's consider an image with pixel values ranging from 50 to 200

Set $Imin = 50$ and $Imax = 200$.

For a pixel value $I(x, y) = 100$

$$I_{enhanced}(x, y) = \frac{100 - 50}{200 - 100} \times (255 - 1) \approx 84.67$$

This transformation adjusts the pixel value from 100 to approximately 85, effectively stretching the contrast.

Example

We can note that:

Before Contrast Stretching: The image may appear dull and lacking detail.

After Contrast Stretching: The image shows improved contrast, making features more distinguishable.

By applying the Gaussian filter and contrast stretching techniques, images can be significantly enhanced for better visual quality and analysis. These methods are foundational in various applications, including medical imaging, photography, and computer vision.

Image restoration

Deconvolution (Wiener Filter):

defined as:

complex conjugate of the signal

the filter

$$H(u,v) = \frac{S^*(u,v)}{|S(u,v)|^2 + K}$$

constant that represents noise

Example: If an image is blurred due to motion, applying the Wiener filter can restore the sharpness by reversing the effects of the blur.

Edge detection and segmentation of images

Edge detection

Canny Edge Detector: The Canny method involves several steps, including gradient calculation:

the gradients in the y direction

$$G(x, y) = \sqrt{I_x^2 + I_y^2}$$

the gradients in the x direction

Example: Using the Canny edge detector on a photograph will highlight the edges of objects within the image, making them more pronounced.

Edge detection and segmentation of images

Image Segmentation

Thresholding: The basic thresholding operation can be expressed as:

$$I_{segmented}(x,y) = \begin{cases} 1 \ if \ I(x,y) > T \\ 0 \ otherwise \end{cases}$$

the threshold value

Example: In segmenting a grayscale image of a fruit, setting a threshold can help isolate the fruit from the background based on intensity.

Wavelet transform

Continuous Wavelet Transform (CWT)

the translation

the scale

$$W(a,b) = \int_{-\infty}^{\infty} f(t)\psi^*(at - b)\,dt$$

the wavelet function

wavelet coefficient

the signal

Example: The wavelet transform can be used for image compression by representing an image in terms of its wavelet coefficients, allowing for efficient storage.

Other Signal and Image Processing Methods

Fourier Transform

Discrete Fourier Transform (DFT)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$X(k)$ → DFT

$x(n)$ → the signal

Example: Applying the DFT to an image allows for frequency analysis, helping to identify periodic patterns or noise.

Other Signal and Image Processing Methods

Morphological Operations

Dilation: The dilation operation can be defined as:

$$(A \oplus B)(x, y) = max_{(i,j)\in B} A(x - i, y - j)$$

A: the image

B: the structuring element.

Example: Dilation can be used to expand the boundaries of objects in a binary image, useful in shape analysis.

Other Signal and Image Processing Methods

Principal Component Analysis (PCA)

PCA Transformation:

$$Z = XW$$

- Z: the transformed data
- X: the original data
- W: the matrix of eigenvectors

Example: PCA can reduce the dimensionality of image data while preserving variance, making it easier to analyze high-dimensional images.

Other Signal and Image Processing Methods

Clustering and Classification

Clustering

K-means Clustering: The objective function can be defined as:

the number of clusters

the data point

$$J = \sum_{i=1}^{k} \sum_{j=1}^{n} \| x_j^{(i)} - \mu_i \|^2$$

the centroid of cluster i

Example: K-means can be applied to segment an image into different color regions based on pixel intensity.

Other Signal and Image Processing Methods

Clustering and Classification

Classification

Support Vector Machine (SVM): The decision boundary can be expressed as:

$$f(x) = w^T x + b$$

- w: the weight vector,
- x: the input feature vector,
- b: the bias

Example: SVM can classify images of handwritten digits by finding the optimal hyperplane that separates different digit classes.

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