Advanced biomedical signal and image processing

Master: Plasturgy & Biomedical Engineering

2025-2026

Faculté de Science Meknes

Professor Omar ELOUTASSI

Section 3 : Processing of Biomedical Images

General Introduction

Chapter 1: X-ray

Chapter 2. Magnetic resonance imaging (MRI)

Chapter 3. Ultrasound imaging

Chapter 4. Nuclear medicine

Chapter 5. Optical imaging

X-Rays

Introduction

Basic physics of X-rays

Absorption and attenuation of X-Rays

Transmission:

Contrast in imaging

Mathematical description

Image formation

Applications

Safety and radiation protection

Introduction

- X-ray imaging is crucial for biomedical engineering students, combining engineering principles and medical applications.
- Students learn how to design, optimize, and use X-ray systems in clinical settings.
- They explore advanced imaging technologies like digital radiography and computed tomography (CT).

Introduction

- Research includes minimizing radiation exposure while maximizing diagnostic accuracy.
- Collaboration with healthcare professionals leads to innovative imaging solutions.
- Engaging with real-world challenges fosters a passion for innovation in healthcare technology.

Basic physics of X-rays

Generalities

Properties

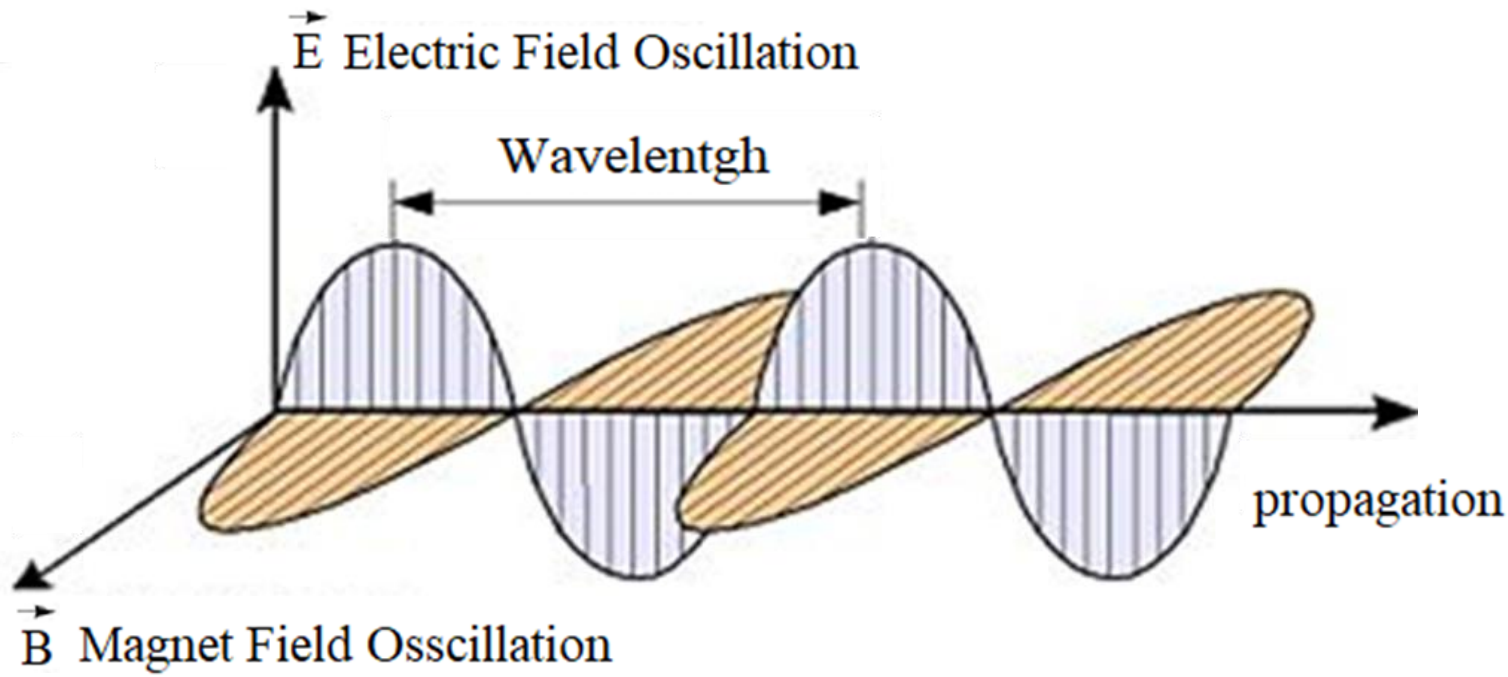
Production of X-Rays

Mechanism of X-Ray production

Interaction of X-Rays with matter

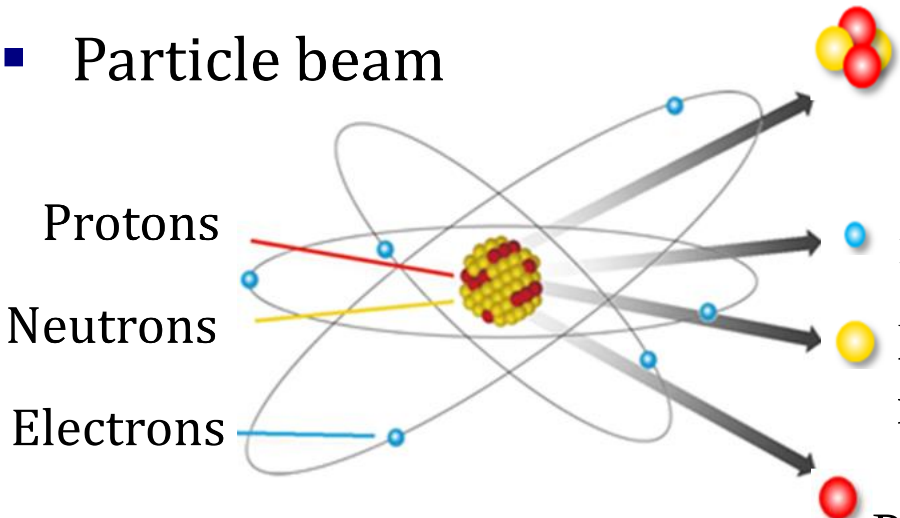
Basic physics of X-rays
Generalities

- A form of electromagnetic radiation



Basic physics of X-rays
Generalities
Ionizing radiation

- Particle beam



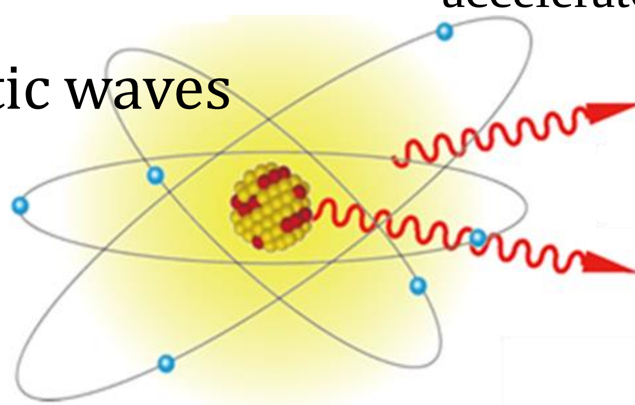
α Particles, helium nuclei ejected from a nucleus

β Particles, electrons ejected from a nucleus

Neutrons beam, produced in nuclear reactors, accelerators, etc.

Proton beams produced in accelerators, etc.

- Electromagnetic waves

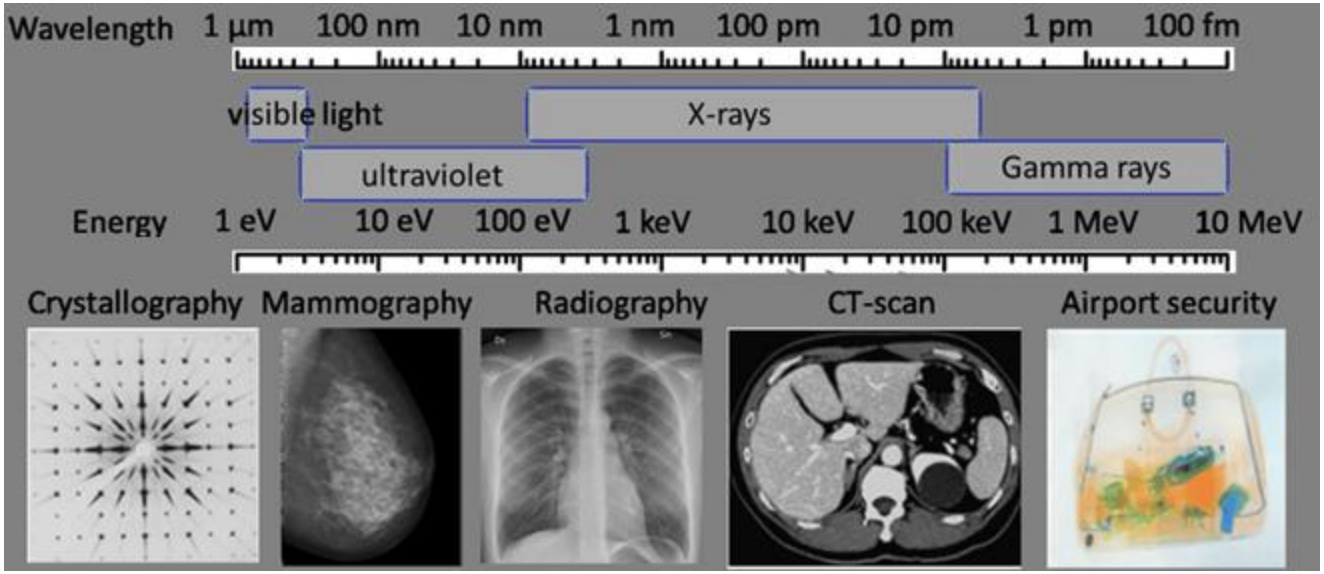


X-Rays, generated outside nucleus

Y-rays, emitted from a nucleus

Basic physics of X-rays
Generalities
Ionizing radiation

- Example of ionising radiation



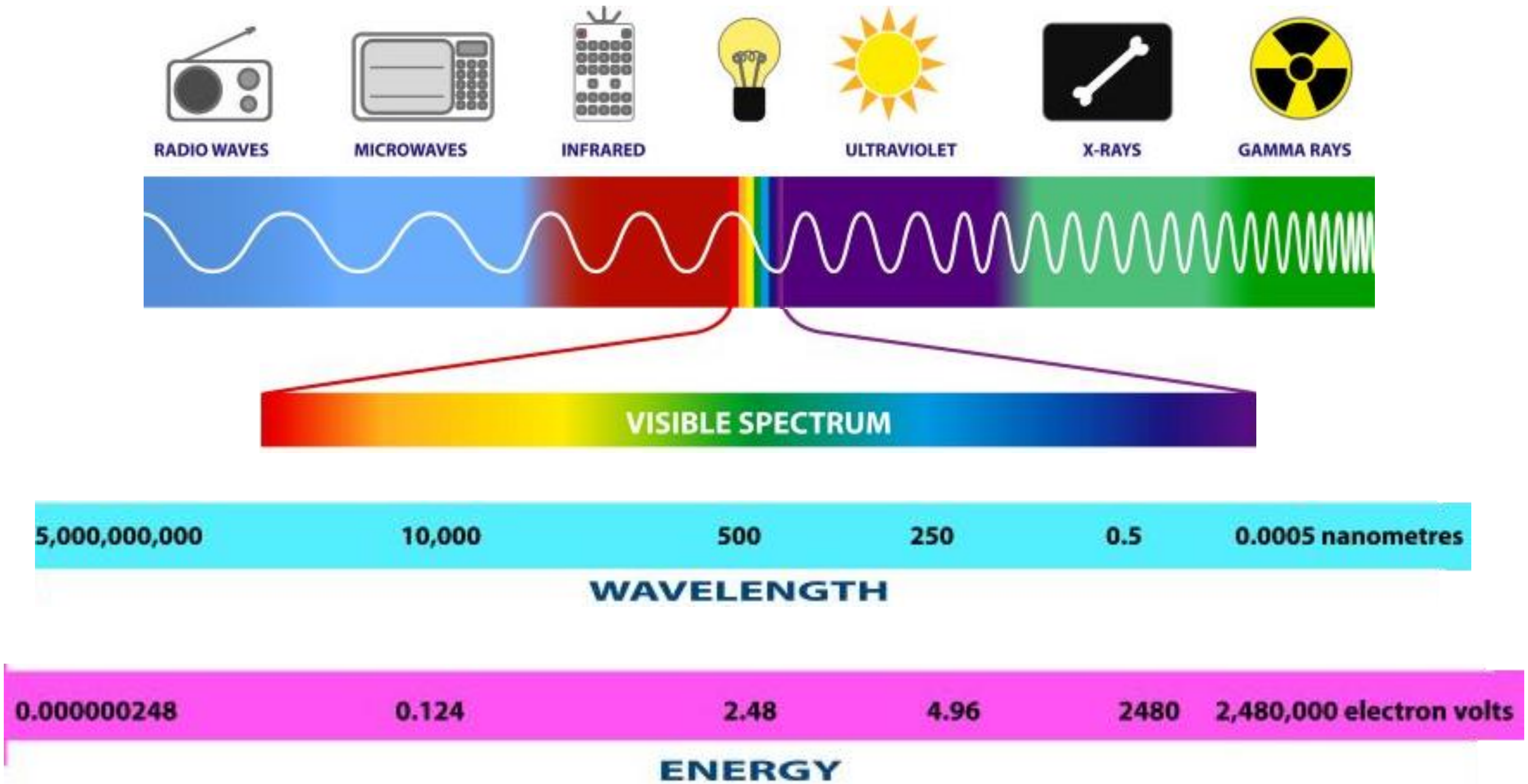
- Frequency
  - $3 \cdot 10^{16}$ Hz – $3 \cdot 10^{19}$ Hz
- Wavelength
  - 0.01 nm – 10 nm
    - Shorter than UV
    - Longer than γ
- Energy
  - 100 eV – 100 keV
    - Higher than UV
    - Lower than γ

Basic physics of X-rays
Properties

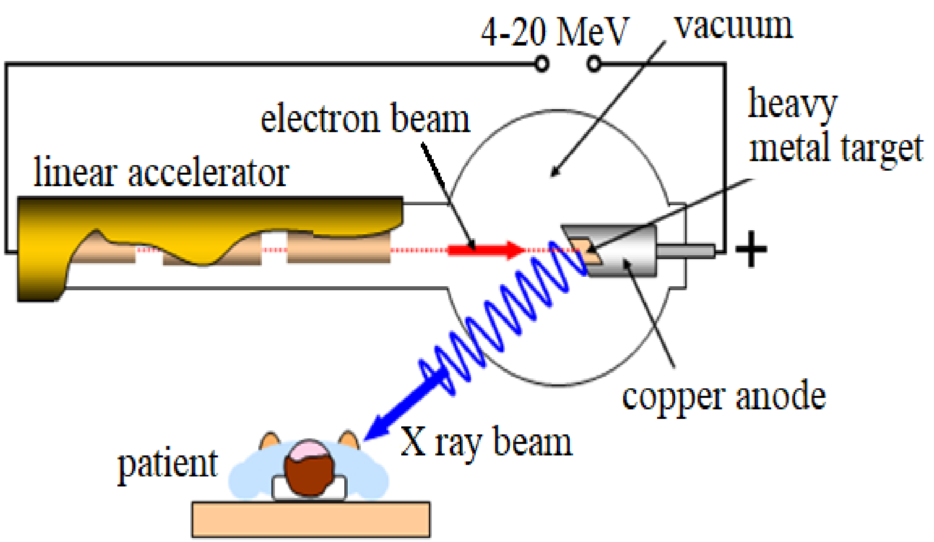
- **Wavelength Range**: ranging from **0.01 to 10nm**.
- **Frequency Range**: 3×10^{15} Hz(PetaHz) to 3×10^{30}Hz (ExaHz).
- $c = \lambda f$

C≈3×10^{8} m/s) speed of light in a vacuum

Basic physics of X-rays
Properties



Basic physics of X-rays
Production of X-Rays



- Particle accelerators
  - Linear accelerator of electrons
    - $E_{electrons} < 50$ MeV
    - Hard X-radiation produced
- X-ray tube
  - Cathode
    - Emits electrons
    - They impact on an anode
      - Focus (spot)
- Efficiency of X-ray production
  - Less than 1% of incident energy
    - The remaining energy is converted into heat
- Maximal intensity of X-rays (Nearly normal to the e^- beam)
- Energy of produced e^- (~ 0.1 MeV)

Basic physics of X-rays
Mechanism of X-Ray production

Thermionic emission: is the process by which electrons are emitted from a material (usually a metal) when it is heated to a high temperature. The number of emitted electrons Richardson equation:

$$I = AT^2 e^{-kT\phi}$$

- I is the current (number of emitted electrons),
- A is the Richardson constant,
- T is the absolute temperature of the cathode in Kelvin,
- ϕ is the work function of the material, which is the minimum energy needed to remove an electron from the surface of the material. A higher work function means that more energy is required for an electron to escape.
- k is the Boltzmann constant,

Basic physics of X-rays
Mechanism of X-Ray production

Acceleration: A high voltage potential (30-150 kV) accelerates electrons toward the anode.

Collision and X-ray generation: High-speed electrons collide with the anode material (usually tungsten), undergo rapid deceleration. This process generates X-ray photons through two main mechanisms:

- **Bremsstrahlung radiation**: This occurs when electrons are deflected by the electric field of the nuclei in the anode material, resulting in the emission of X-ray photons.

The energy of the emitted photons

$$E = eV$$

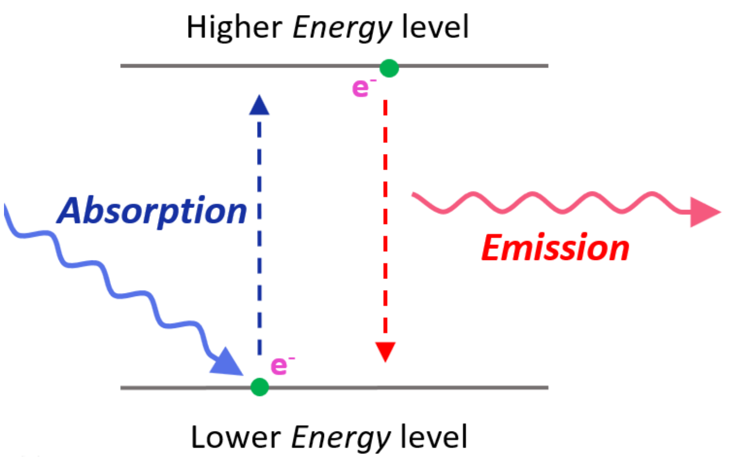
E is the energy of the X-ray photon

e is the charge of the electron

V is the accelerating voltage.

Basic physics of X-rays
Mechanism of X-Ray production

- **Characteristic Radiation**: An incoming electron knocks out an inner-shell electron from the anode material, causing an outer-shell electron to fall into the vacancy, emitting an X-ray photon with energy characteristic of the anode material.



The energy of the characteristic X-ray

$$E = Eshell1 - Eshell2$$

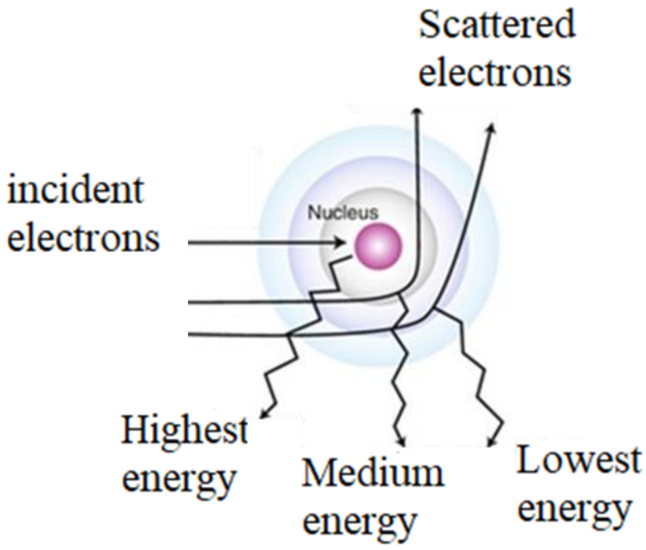
$Eshell1$ is the energy of the higher energy shell

$Eshell2$ is the energy of the lower energy shell.

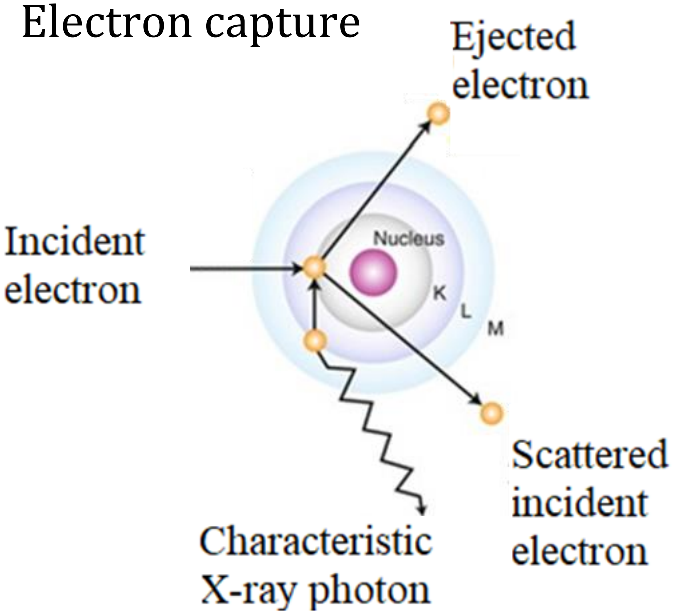
Basic physics of X-rays
Mechanism of X-Ray production

- X-rays are generated
  - Whenever electrons become stopped after striking a heavy metal target with a sufficiently high velocity
  - Through two different atomic processes

- Bremsstrahlung
  - Braking radiation



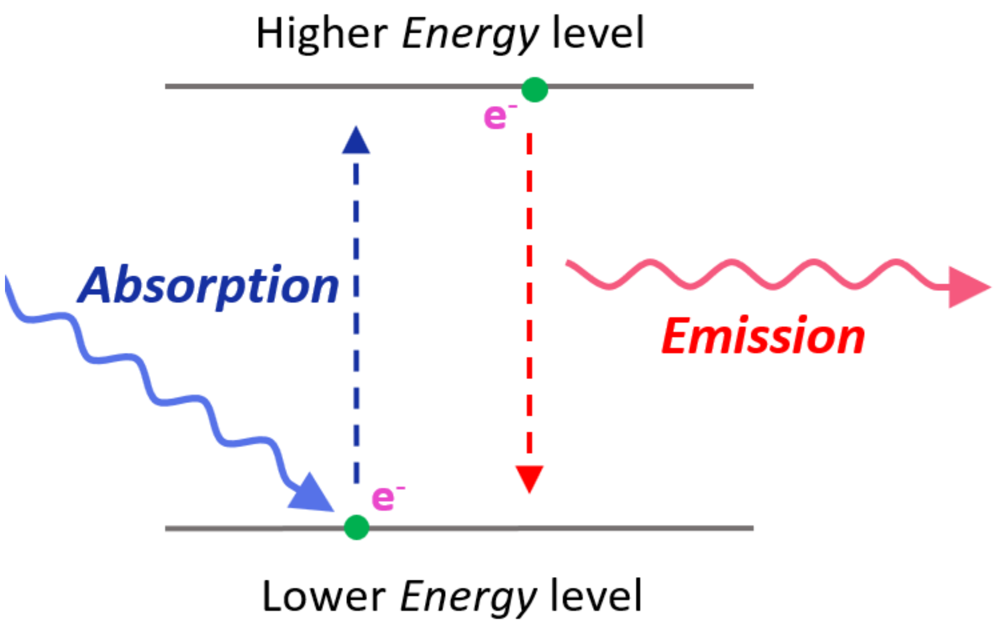
- K-shell emission
  - Electron capture



Basic physics of X-rays
Interaction of X-Rays with matter

When X-rays pass through matter, they can interact in several ways:

Photoelectric Effect: An X-ray photon is completely absorbed by an atom, resulting in the ejection of an inner-shell electron. This effect is significant at lower energies and for high atomic number materials.



Basic physics of X-rays
Interaction of X-Rays with matter

Compton Scattering: An X-ray photon collides with a loosely bound outer-shell electron, resulting in partial energy transfer. The photon is scattered at a lower energy and different angle.

The energy and angle

$$E' = \frac{E}{1 + \frac{E}{m_e c^2 (1 - cos\theta)}}$$

E	energy of the incoming photon
E'	energy of the scattered photon,
c	speed of light,
me	the mass of the electron,
θ	the scattering angle.

Basic physics of X-rays
Interaction of X-Rays with matter

Rayleigh Scattering: Elastic scattering occurs without energy loss, primarily at low energies and in small particles.

Pair Production: At very high energies (greater than 1.022 MeV), an X-ray photon can produce an electron-positron pair when interacting with a nucleus.

Absorption and attenuation of X-Rays:

Photoelectric Effect

Contrast enhancement

Compton scattering

Absorption and attenuation of X-Rays: Photoelectric Effect

The **photoelectric effect** occurs when an X-ray photon is completely absorbed by an atom, resulting in the ejection of an inner-shell electron.

Energy Dependence: The probability of the photoelectric effect occurring is highly dependent on the energy of the X-ray photon (E) and the atomic number (Z) of the absorbing material.

The photoelectric absorption coefficient

$$\mu_{PE} \propto \frac{Z^3}{E^3}$$

higher atomic number materials (like lead) are more effective at absorbing X-rays, enhancing image contrast.

Absorption and attenuation of X-Rays: Photoelectric Effect

- **Threshold Energy**: The minimum energy required to eject an electron from an inner shell is given by the work function (ϕ) of the material.

- The incident photon must satisfy:

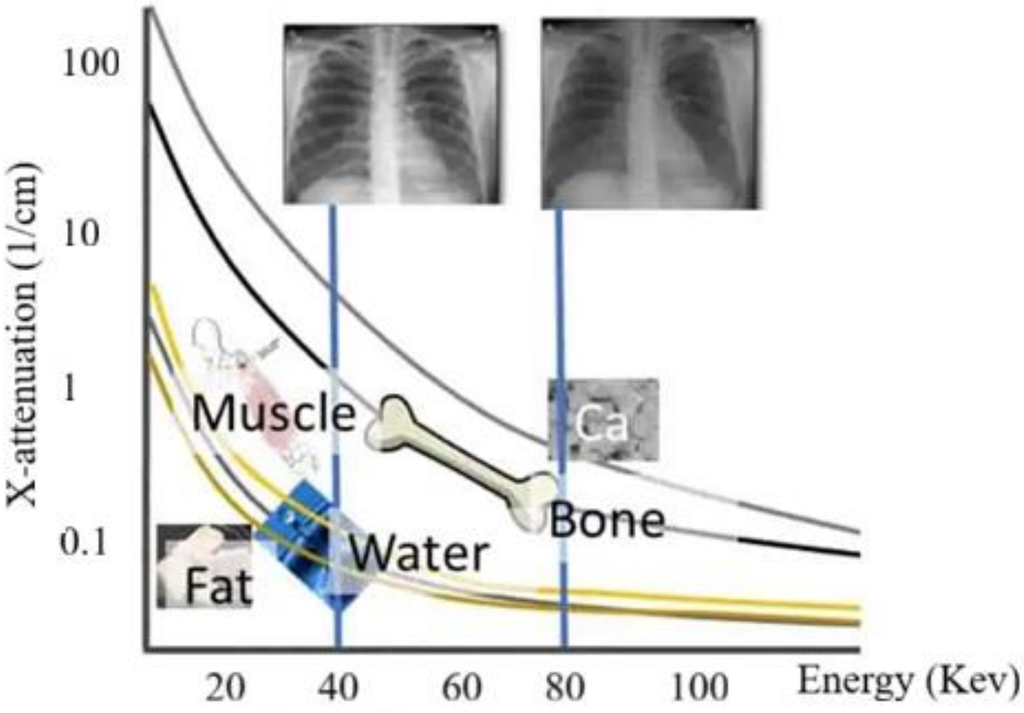
$$E \geq \phi$$

E is the energy of the incoming X-ray photon.

Absorption and attenuation of X-Rays
Contrast enhancement

The increased absorption in high-Z materials leads to greater differences in attenuation between different tissues,

crucial for producing clear images.



Absorption and attenuation of X-Rays: Contrast enhancement

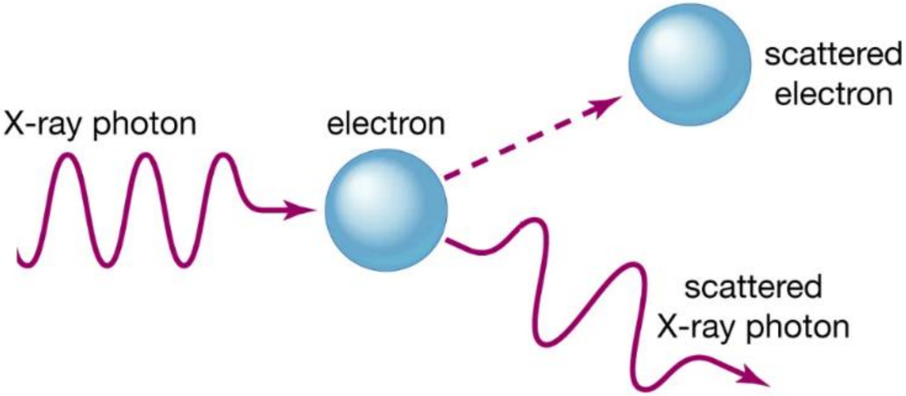
The contrast (C) in an X-ray image can be quantified as:

$$C = \frac{I_1 - I_2}{I_1 + I_2}$$

I_1 and I_2 are the intensities of X-rays transmitted through different tissues.

Absorption and attenuation of X-Rays: Compton scattering

Occurs when X-ray photons interact with loosely bound outer-shell electrons resulting in a reduction in energy (longer wavelength) and a change in direction



This scattering contributes to image noise and reduces overall image contrast.

Absorption and attenuation of X-Rays: Compton scattering

The energy of the scattered photon

$$E' = \frac{E}{1 + \frac{E}{m_e c^2 (1 - cos\theta)}}$$

E energy of the incident photon,

E' energy of the scattered photon,

me mass of the electron (9.11×10^{-31}kg),

c speed of light (3×10^{8} m/s),

θ scattering angle.

Absorption and attenuation of X-Rays: Compton scattering

The change in wavelength ($\Delta\lambda$) due to Compton scattering

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - cos\theta)$$

λ is the initial wavelength,

λ' is the wavelength after scattering,

h is Planck's constant (6.626×10^{-34} J s).

Absorption and attenuation of X-Rays: Compton scattering

Impact on Image Quality: The scattered photons can contribute to image noise, as they can scatter in various directions, reducing the clarity of the image.

The overall attenuation

$$\mu_C = \mu_{C0}.\, \rho.\, \frac{E_0}{E}$$

μ_{C0}, mass attenuation coefficient for Compton scattering,

ρ density of the material,

E_0 initial energy of the photon,

E energy after scattering.

Transmission:

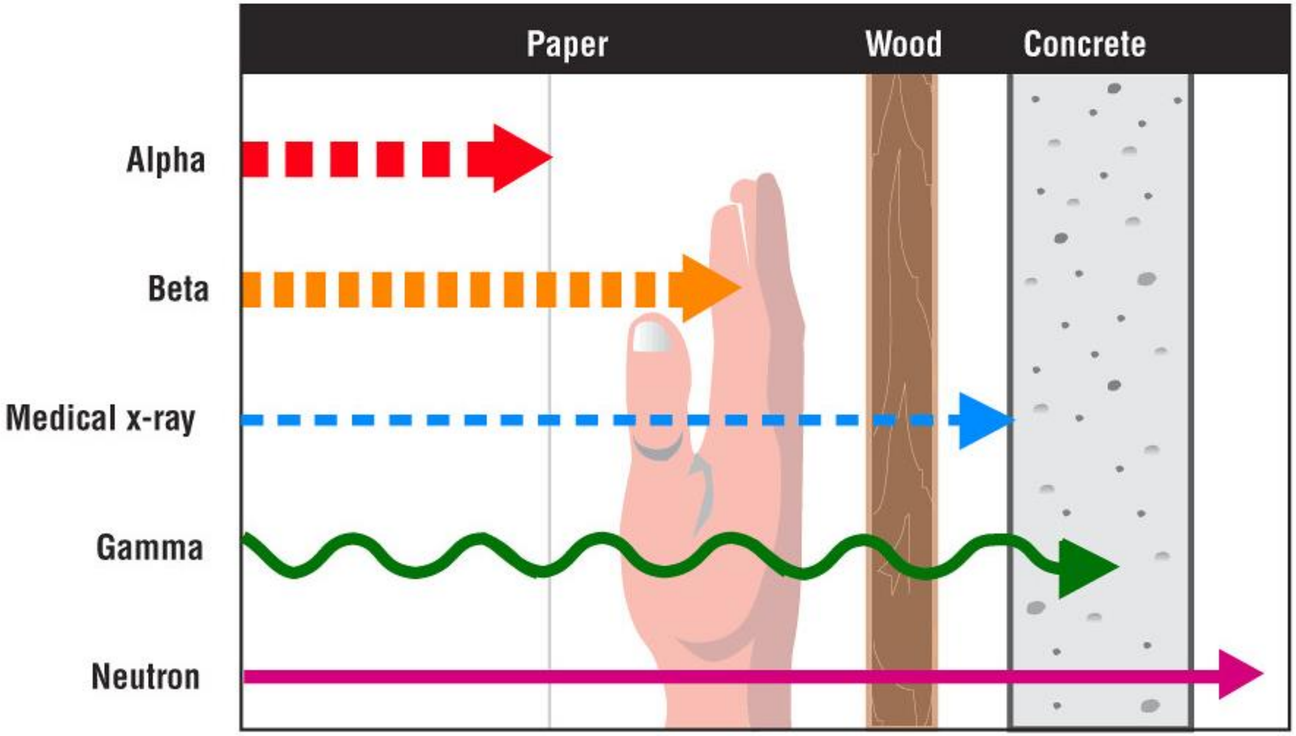
Penetration of X-Rays

Attenuation and half-value layer

Transmission:

Penetration of X-Rays

X-rays have the ability to penetrate various materials, including human tissues.



Transmission:

Penetration of X-Rays

The extent of penetration is influenced by several factors

- **Energy of X-Rays**: Higher energy X-rays have greater penetrating power.

  $E = hf$

  E is the energy of the photon,

  h is Planck's constant (6.626×10^{-34} Js),

  f is the frequency of the X-ray.

As the frequency increases (wavelength decreases), the energy of the X-ray photons increases, allowing them to penetrate more dense materials.

Transmission:

Penetration of X-Rays

Material density: The density (ρ) of the material also plays a crucial role in determining how much X-ray radiation is absorbed or transmitted.

Denser materials have more atoms per unit volume, which increases the likelihood of interactions with X-ray photons.

Composition of material: The atomic number (Z) of the elements in the material significantly affects X-ray penetration.

Materials with higher atomic numbers (like lead) are more effective at absorbing X-rays due to increased photoelectric absorption and Compton scattering.

Transmission:

Attenuation and half-value layer

The **attenuation** of X-rays as they pass through a material

$$I = I_0 e^{-\mu x}$$ Beer-Lambert law

I intensity of the X-ray after passing through a distance x,

I_0 initial intensity of the X-ray,

μ linear attenuation coefficient of the material, which depends on both the energy of the X-rays and the material properties.

Transmission:

Attenuation and half-value layer

The **linear attenuation coefficient** (μ) can be expressed as:

$$\mu=\mu_{PE}+\mu C+\mu_R$$

μ_{PE} is the contribution from the photoelectric effect,

μ_C is the contribution from Compton scattering,

μ_R is the contribution from Rayleigh scattering.

Transmission:

Attenuation and half-value layer

The **half-value layer (HVL)** is defined as the thickness of a material required to reduce the intensity of X-rays to half its original value. It is calculated from the linear attenuation coefficient:

$$HVL = \frac{ln(2)}{\mu}$$

$\ln(2) \approx 0.693$.

The HVL provides a useful measure of the penetrating ability of X-rays through different materials.

Contrast in imaging
Differential absorption

Differential absorption refers to the varying degrees to which different tissues in the body absorb X-rays.

Density of Tissues: The density (ρ) of tissues plays a significant role in how much X-ray radiation is absorbed.

Atomic Number: The atomic number (Z) of the elements in the tissue also affects absorption.

Contrast in imaging
Absorption coefficients

The **linear attenuation coefficient** (μ) quantifies how much X-ray intensity decreases as it passes through a material. It is influenced by both the energy of the X-rays and the composition of the material. The relationship can be expressed as we described above.

Watch this video
https://www.youtube.com/watch?v=wwALmYKUZkc&t=461s

Mathematical description

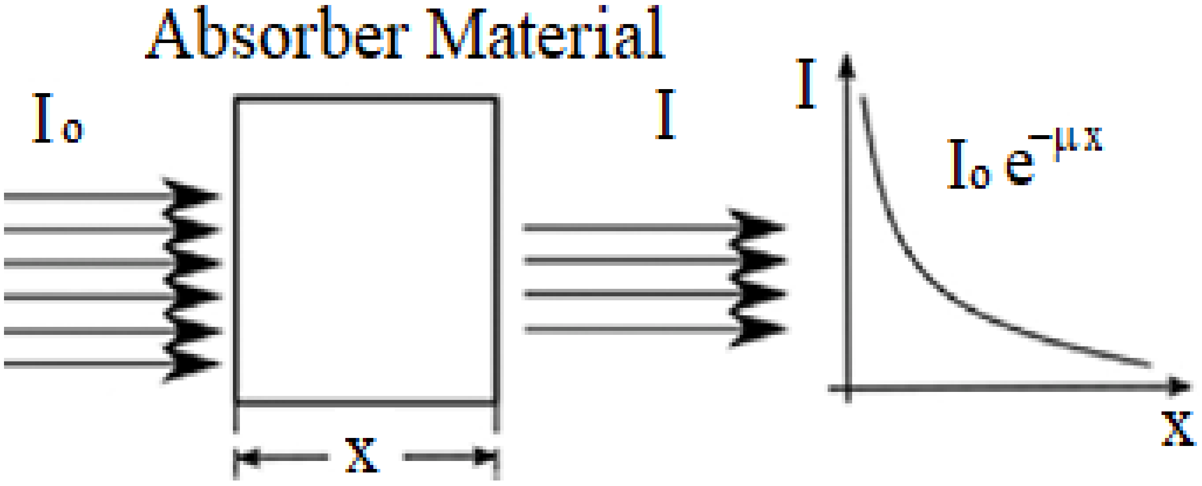
Beer-Lambert law

Applications in medical imaging

Limitations

Mathematical description
Beer-Lambert law

The **Beer-Lambert law** is a fundamental principle that describes how the intensity of light (or other electromagnetic radiation, including X-rays) decreases as it passes through a medium.



Mathematical description
Beer-Lambert law

Transmitted intensity (I): This is the amount of X-ray intensity that emerges from the material after interacting with it. It reflects how much of the incident X-ray beam has been absorbed or scattered.

Initial intensity ($I0$): This represents the intensity of the X-ray beam before it interacts with the material. It is the reference point for measuring attenuation.

Linear attenuation coefficient (μ): This coefficient quantifies how easily a material can attenuate X-rays. It is influenced by:

The material's density (ρ): Denser materials generally have higher attenuation coefficients.

Mathematical description
Beer-Lambert law

The atomic number (Z): Materials with higher atomic numbers tend to absorb X-rays more effectively due to increased interactions (photoelectric effect and Compton scattering).

The energy of the X-rays: Higher energy X-rays may penetrate materials more easily, resulting in lower attenuation coefficients.

Thickness of Material (x): The distance the X-rays travel through the material directly affects the extent of attenuation. Thicker materials will result in greater absorption and scattering.

Mathematical description
Applications in medical imaging

In medical imaging, the Beer-Lambert Law helps in understanding how different tissues absorb X-rays. For instance:

- **Bone vs. soft tissue**: Bones, being denser and containing higher atomic number elements (like calcium), have a higher linear attenuation coefficient compared to soft tissues,
- **Quantitative analysis**: The Beer-Lambert Law allows radiologists to estimate the thickness and density of tissues based on the intensity of X-rays that are transmitted. By analyzing the intensity of the X-ray beam before and after it passes through a patient, medical professionals can infer important diagnostic information.

Mathematical description
Limitations

While the Beer-Lambert Law is widely applicable, it has some limitations, especially in **complex biological tissues** where multiple scattering events and **non-linear effects may occur**. Additionally, it assumes a uniform medium and does not account for **variations in tissue composition** or structure.

Image formation:

Projection imaging

Image formation process

Image formation:

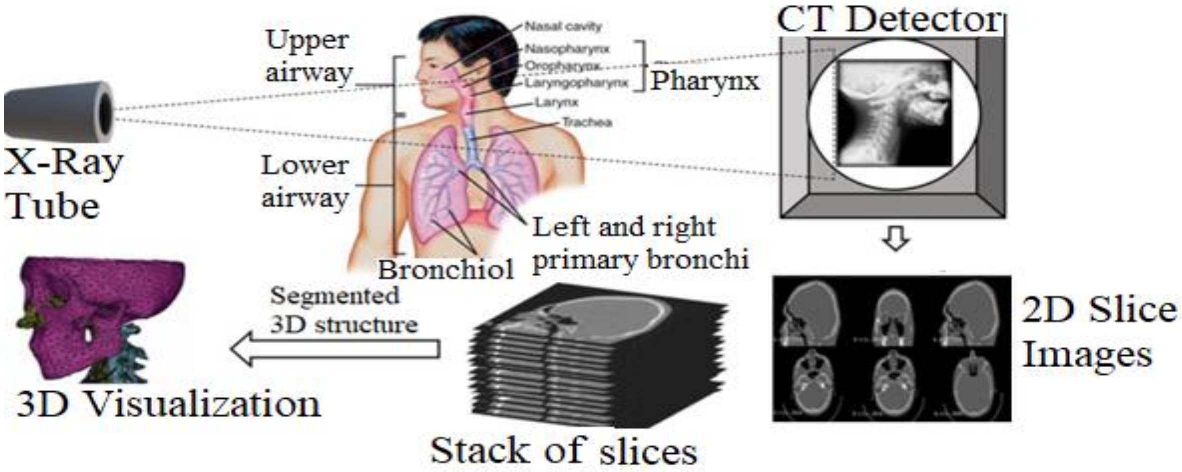
Projection imaging

In **conventional radiography**, a **2D image** is formed by projecting a **3D structure** onto a detector.
This process involves many key concepts

- **X-ray generation**: X-rays are produced by the interaction of high-energy electrons with a target material (usually tungsten) in the X-ray tube. The energy of the emitted X-ray photons is typically in the range of 20-150 keV.
- **Path of X-ray beam**: As X-rays pass through the body, they interact with various tissues, undergoing **absorption** and **scattering**. The intensity of the X-rays reaching the detector depends on the cumulative absorption along the path of the X-ray beam.
- **Cumulative absorption**: The intensity of the X-rays that reach the detector can be described using the Beer-Lambert Law.

Image formation:
Image formation process

The formation of a radiographic image involves the following steps:



➢**Emission**: X-rays are emitted from the tube and directed towards the patient.

➢**Tissue interaction**: X-rays pass through various tissues, with different degrees of absorption based on the tissue density and atomic number.

➢**Detection**: The remaining X-rays that pass through the body strike a detector (film or digital), forming a latent image.

➢**Image processing**: The latent image is processed to produce a visible image, highlighting the differences in absorption between various tissues.

Applications

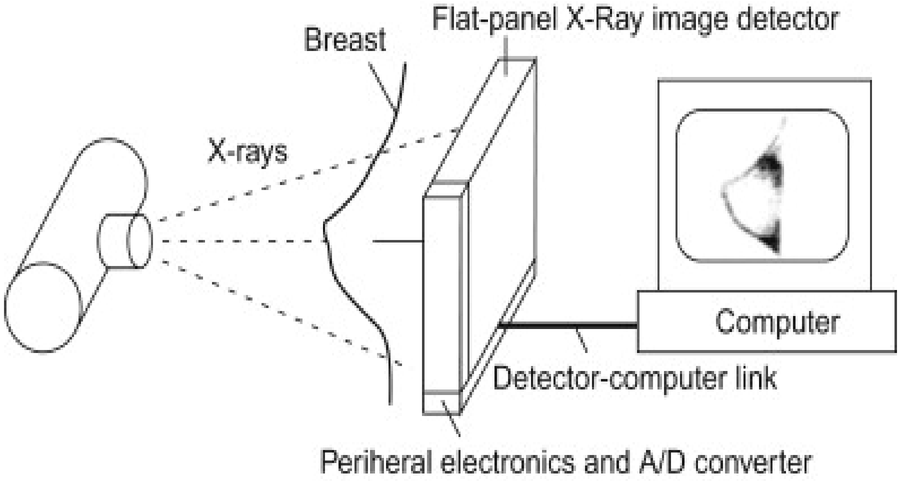
Modern digital detectors

Specialized techniques

Safety and radiation protection

Applications
Modern digital detectors

- **Digital conversion**: Modern digital detector systems convert X-ray photons directly into digital signals. This conversion allows for enhanced image processing, storage, and retrieval, significantly improving diagnostic capabilities.



- **Reduced radiation dose**: Digital systems often require lower doses of radiation compared to traditional film-based systems. This is due to their higher sensitivity and the ability to manipulate image contrast and brightness digitally.

Applications
Specialized techniques

CT Scanning (computed tomography):

- **Mechanism**: CT scans use X-rays taken from multiple angles around the body to create detailed cross-sectional images. The mathematical reconstruction of these images is typically done using algorithms like the **Filtered back projection** or **iterative reconstruction**.
- **Mathematical representation**: The image reconstruction can be expressed in terms of the Radon transform, which relates the projection data to the original image function.

Applications
Specialized techniques

Fluoroscopy:

➢**Real-time imaging**: This technique provides real-time moving images of the internal structures of the body, making it useful for guiding procedures such as catheter insertions.

➢**Continuous X-ray exposure**: Fluoroscopy involves continuous exposure to X-rays, necessitating careful management of radiation dose.

Applications
Specialized techniques

Mammography:

➢**Specialized imaging**: Mammography is a specialized form of X-ray imaging designed specifically for breast tissue. It uses lower radiation doses and higher contrast to detect abnormalities.

➢**Screening and diagnosis**: It is crucial for early detection of breast cancer, often employing techniques like digital mammography or tomosynthesis.

Applications
Safety and radiation protection

Radiation dose management

Healthcare professionals must carefully manage radiation doses because X-rays are a form of ionizing radiation that can damage living tissues. To minimize exposure and avoid adverse effects such as:

- ➢**Radiation burns**
- ➢**Radiation sickness**
- ➢**Increased cancer risk**

Applications
Safety and radiation protection

Protection Measures

To safeguard patients and healthcare workers from unnecessary exposure, several protection measures should be respected:

- **Thyroid shields**: Protect the thyroid gland from scatter radiation during X-ray procedures.
- **Lead aprons**: Use lead aprons to shield other parts of the body.
- **Distance and shielding**: Maintain distance from the radiation source and use barriers to reduce exposure.

END