Advanced biomedical signal and image processing

Master: Plasturgy & Biomedical Engineering

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Faculté de Science Meknes

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Section 3 : Processing of Biomedical Images

Nuclear Medicine

- **INTRODUCTION**
- **MATHEMATICAL CONCEPTS**
- **Image Reconstruction in Nuclear Medicine**
- **Dosimetry**

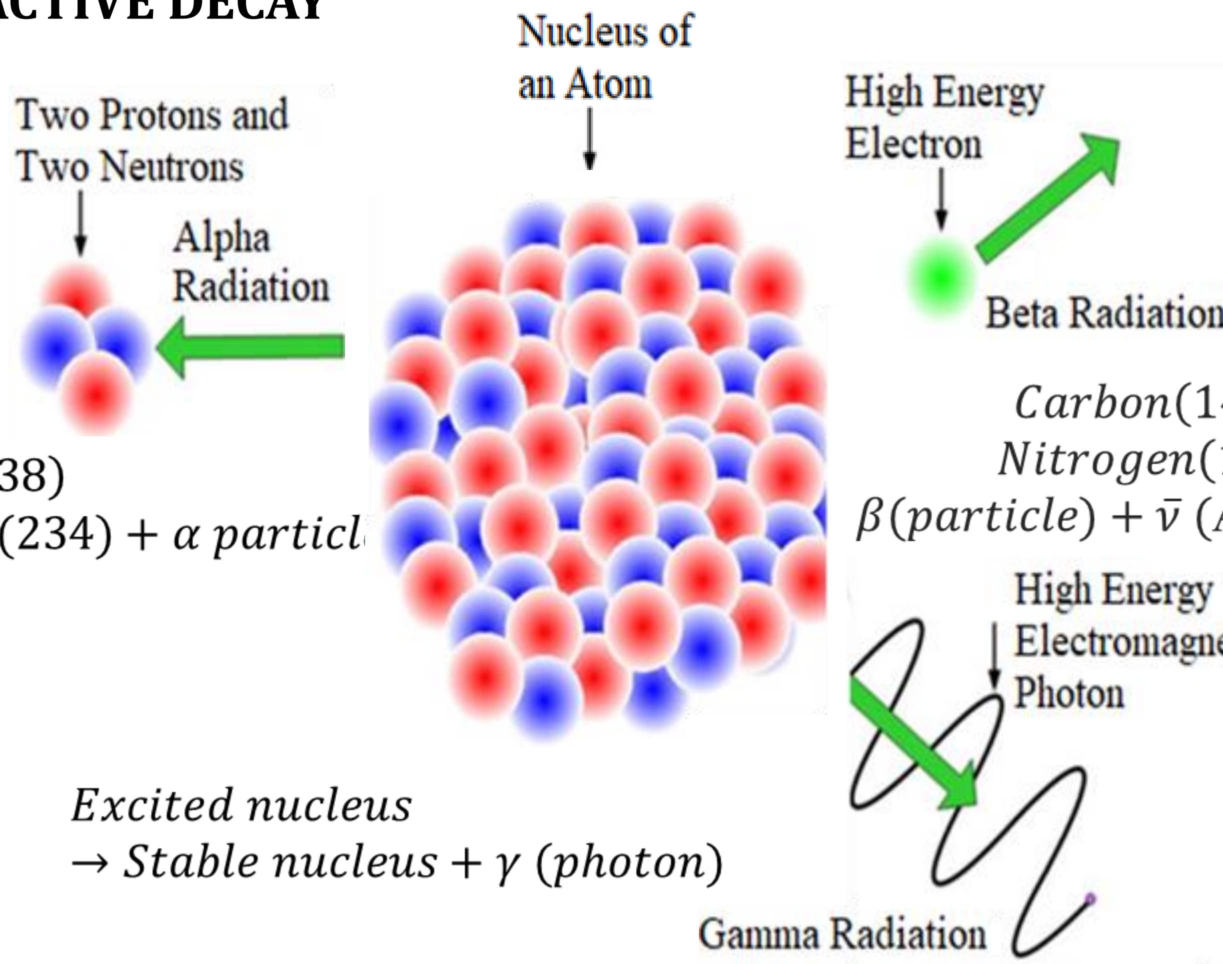
Introduction

- The primary mathematical concepts in nuclear medicine relate to the physics of radioactive decay
- interaction of radiation with matter, the modelling of biological processes, and the reconstruction of images from the detected radiation.
- The use of small amounts of radioactive material, a special camera, and a computer help to create images of the inside of the body.
- Nuclear medicine provides information that cannot be obtained using other imaging methods.
  - It helps diagnose many types of cancers
  - heart disease
  - gastrointestinal, endocrine
  - neurological disorders
  - detect disease in its earliest stages

To be watch: https://www.youtube.com/watch?v=Ab8eLQbfCig&t=1s

MATHEMATICAL CONCEPTS

- **RADIOACTIVE DECAY**



$Uranium(238)$
$\rightarrow Thorium(234) + \alpha\ particl$

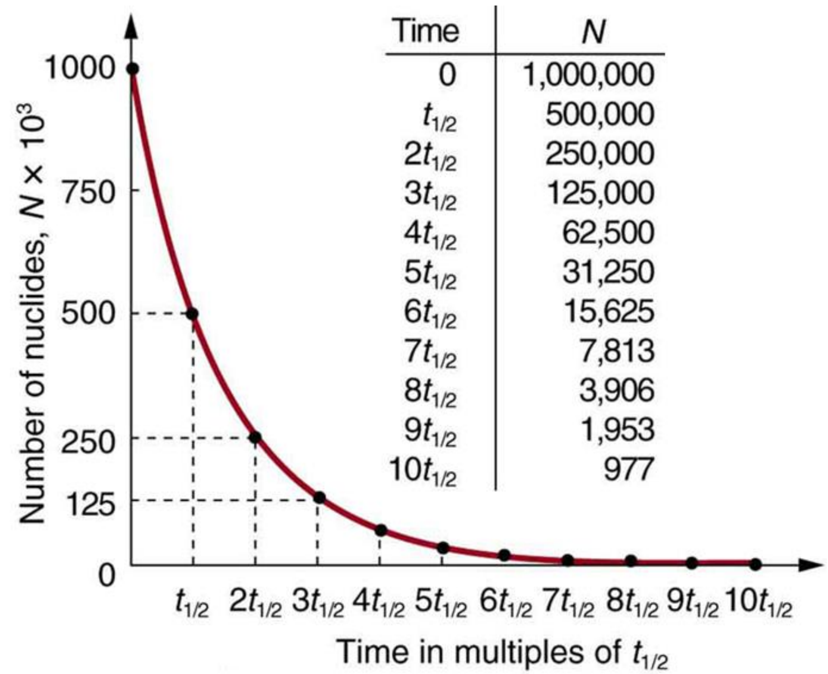
$Carbon(14) \rightarrow$
$Nitrogen(14) +$
$\beta(particle) + \bar{\nu}$ (Antineutrino)

$Excited\ nucleus$
$\rightarrow Stable\ nucleus + \gamma\ (photon)$

MATHEMATICAL CONCEPTS

DECAY LAW

- Radioactive nuclei number $N(t) = N_0 e^{-\lambda t}$ decreases over time
  - N_0 initial nuclei number.
  - λ decay constant (specific to each radionuclide)



Activity

- The activity $A(t) = \lambda N(t) = A_0 e^{-\lambda t}$ radioactive rate decay, measured in Becquerel (Bq) or Curies (Ci).

  A_0 initial activity

Half-life

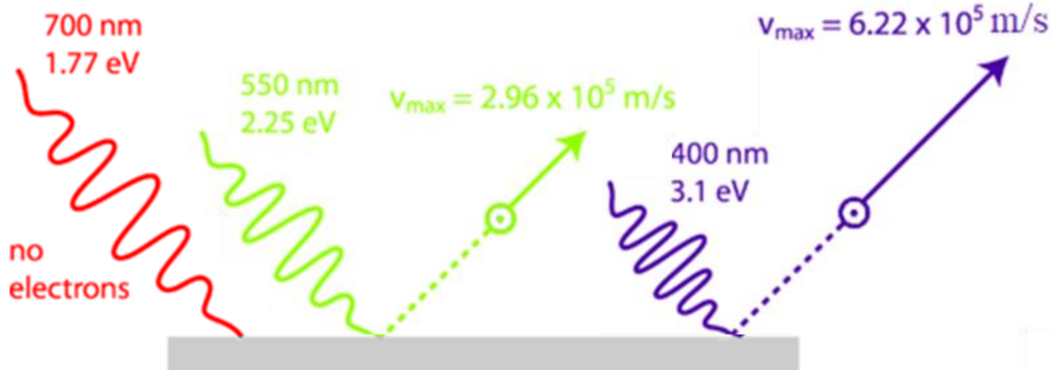
- The half-life $(T_{1/2})$ is the time it takes for half of the radioactive nuclei to decay: $T_{1/2} = \dfrac{\ln(2)}{\lambda}$

MATHEMATICAL CONCEPTS

- **INTERACTION OF RADIATION WITH MATTER**
- **Photoelectric effect**

$E_{photon} = h\nu$

Potassium requires 2.0 eV to eject an electron



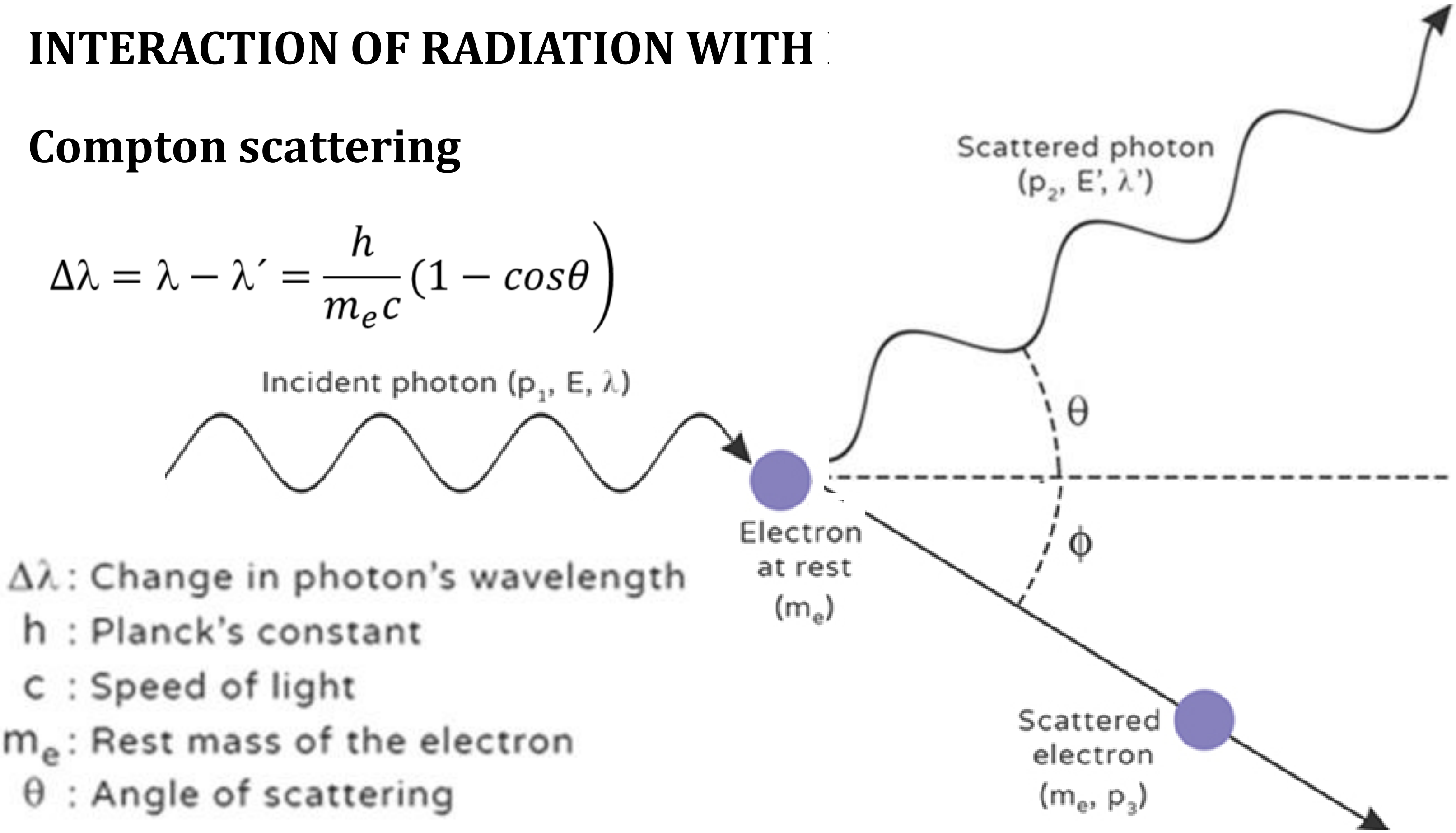
Low frequency light (red) is unable to cause ejection of electrons from the metal surface.
Above the threshold frequency (green) electrons are ejected.
higher frequency incoming light (blue) causes ejection of the same number of electrons, but with greater speed

MATHEMATICAL CONCEPTS

- **INTERACTION OF RADIATION WITH**

- **Compton scattering**

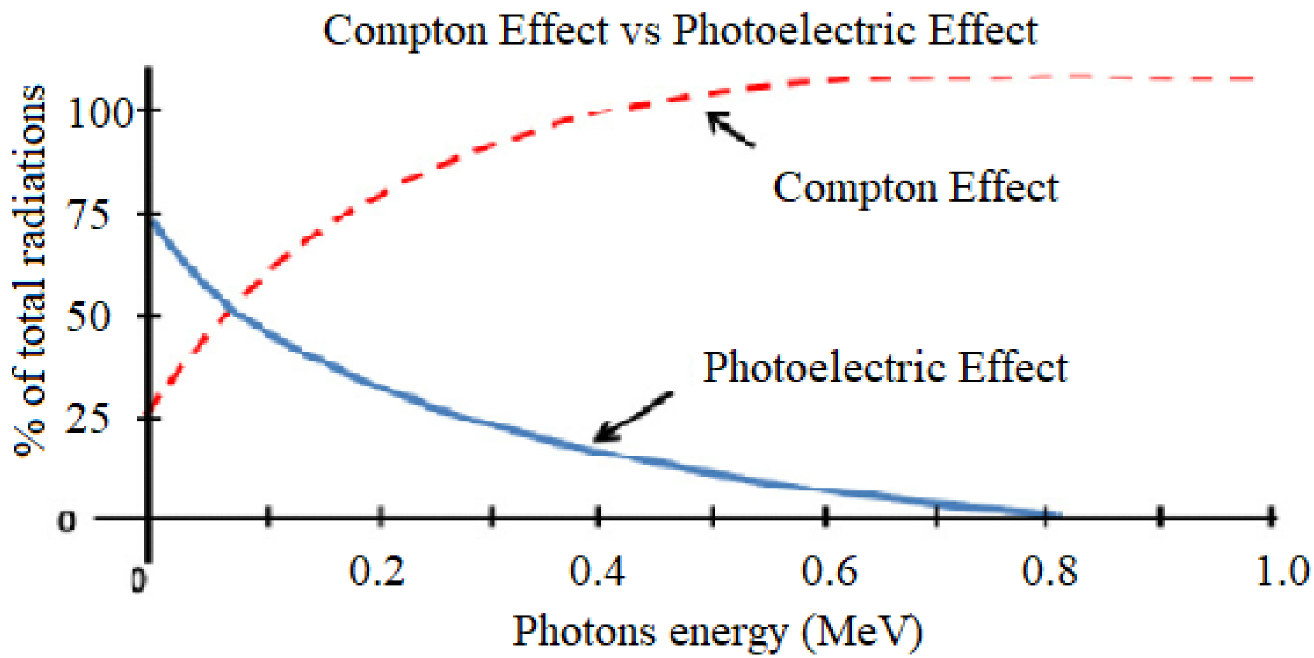
$$\Delta\lambda = \lambda - \lambda' = \frac{h}{m_e c}(1 - cos\theta\Big)$$



MATHEMATICAL CONCEPTS

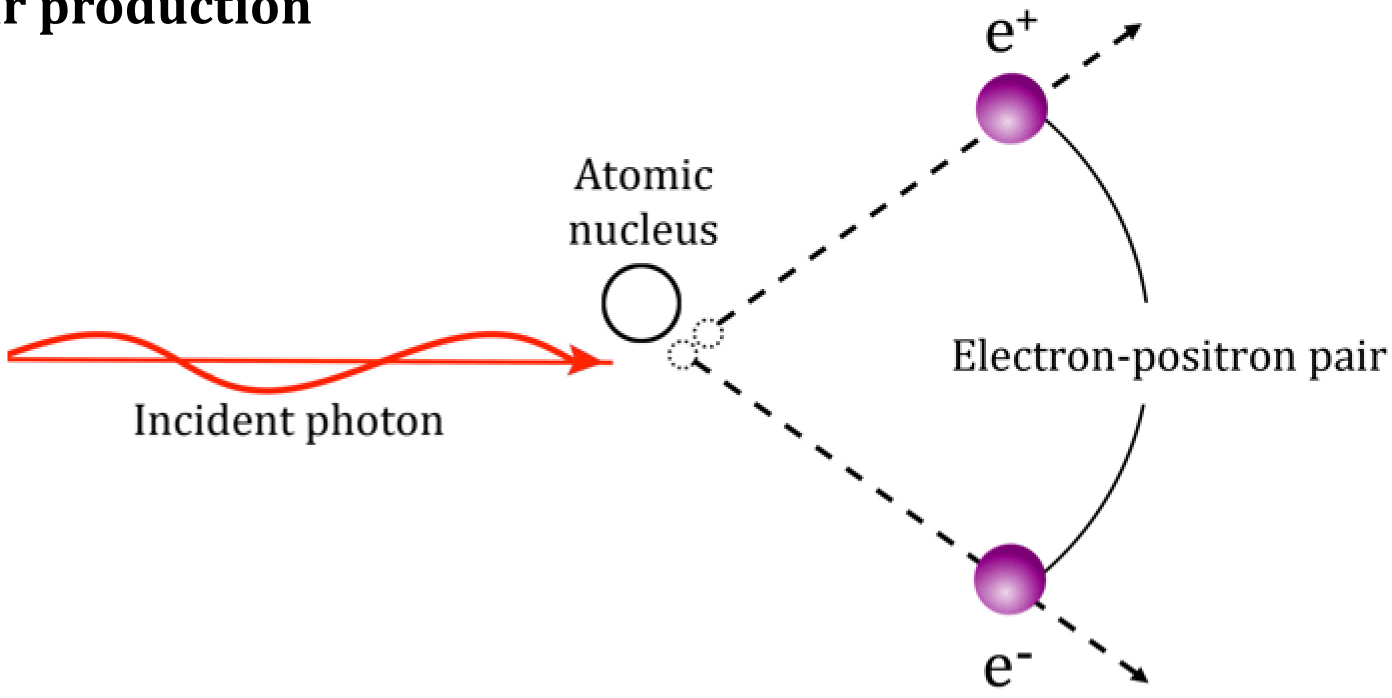
- **Attenuation**

Exponential attenuation



MATHEMATICAL CONCEPTS

- **INTERACTION OF RADIATION WITH MATTER**
- **Pair production**



MATHEMATICAL CONCEPTS

- **Attenuation**

Exponential attenuation

when gamma rays or other types of radiation pass through a medium, their intensity decreases exponentially

$$I(x) = I_0 e^{-\mu x}$$

μ: linear attenuation coefficient, dependent on the material and the energy of the radiation.

I_0: initial intensity

$I(x)$: intensity after traveling a distance x through the medium

MATHEMATICAL CONCEPTS

- **Attenuation**

Exponential attenuation

Applications

Radiometric Dating: Used in archaeology and geology to date ancient artifacts and geological formations based on the known decay rates of isotopes like Carbon-14 and Uranium-238.

Nuclear Medicine: Understanding the decay of radiopharmaceuticals helps in determining appropriate dosages and timing for diagnostic imaging and treatment.

Radiation Safety: Knowledge of decay rates is critical for managing exposure to radioactive materials and ensuring safety in environments where radiation is present.

MATHEMATICAL CONCEPTS

- **Attenuation**

Exponential attenuation

- Activity

radioactive sample is the rate of decay, measured in Becquerel (Bq) or Curies (Ci). It is related to the number of nuclei (N)

Units: The activity is measured in Becquerel (Bq = one decay per second, or in Curies (Ci),

$$1\,Ci = 3.7 \times 10^{10} Bq.$$

$$A(t) = \lambda N(t) = A_0 e^{-\lambda t}$$

- $A(t)$: activity at time t
- A_0: initial activity at time $t = 0$
- λ: radioactive decay constant (probability of a nucleus decaying per unit of time)
- $N(t)$: number of radioactive nuclei at time t
- $e^{-\lambda t}$: fraction of the initial number of radioactive nuclei that remain at time t.

MATHEMATICAL CONCEPTS

- **Radiopharmaceutical kinetics**

- **Compartmental Models**

One compartmental model $C(t) = C_0 \cdot e^{-k \cdot t}$ concentration of the substance at time t

C_0 initial concentration

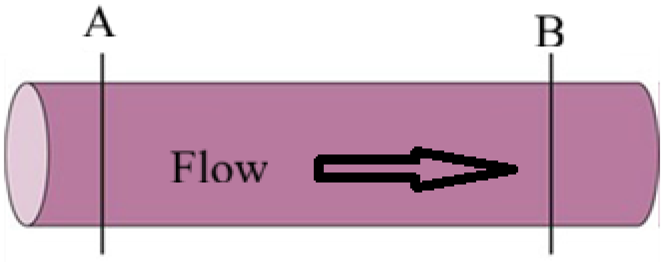
k elimination rate constant

Two compartmental model $C(t) = C(t) = A \cdot e^{-\alpha t} + B \cdot e^{-\beta t}$

A and B are coefficients related to the distribution and elimination phases respectively

α and β are the rate constants for distribution and elimination phases, respectively

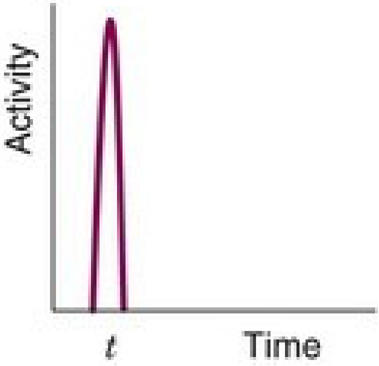
IMAGE RECONSTRUCTION



Tracer kinetics

- **Linear Model** $C(t) = C_0 e^{-\lambda t}$

when using a tracer dose (a small amount of radiopharmaceutical), the kinetics can often be assumed to be linear



- **Non-linear Models** $C(t) = C(t) = A \cdot e^{-\alpha t} + B \cdot e^{-\beta t}$

where higher doses are used or in more complex systems, non-linear models may be necessary.

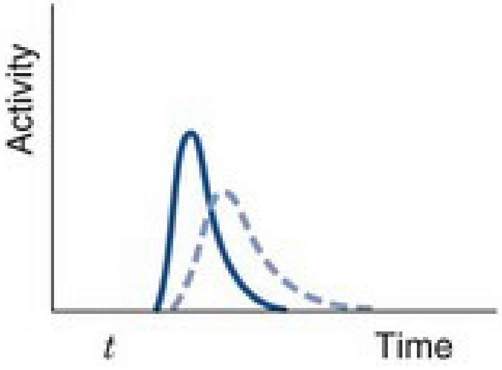


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IMAGE RECONSTRUCTION

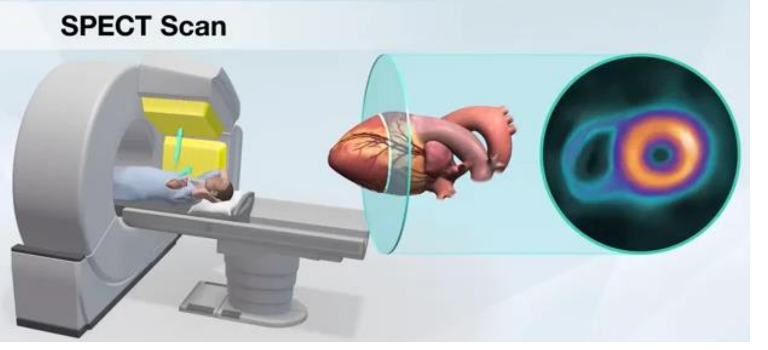
SPECT (Single photon emission computed tomography):

- **Projection data**: SPECT involves acquiring projection data at multiple angles. Each projection can be represented mathematically as:

$$P_\theta(x') = \int_{-\infty}^{\infty} \rho(x,y)\,dy$$

$P_\theta(x')$: projection at angle θ along line x'

$\rho(x,y)$: radionuclide distribution in the object



- **Filtered back projection (FBP)**

A common method for reconstructing the 2D image from the projections.

To be watch: https://www.youtube.com/watch?v=lTBpPYWZcW8

The reconstructed image

$$\rho(x,y) = \int_0^{\pi}\int_{-\infty}^{\infty} [P_\theta(x')\,(h(x' - x\cos(\theta) - y\sin(\theta))dx']d\theta'$$

h: a filter function

IMAGE RECONSTRUCTION

To be watch: https://www.youtube.com/watch?v=P0evlYUY5wo&t=836s

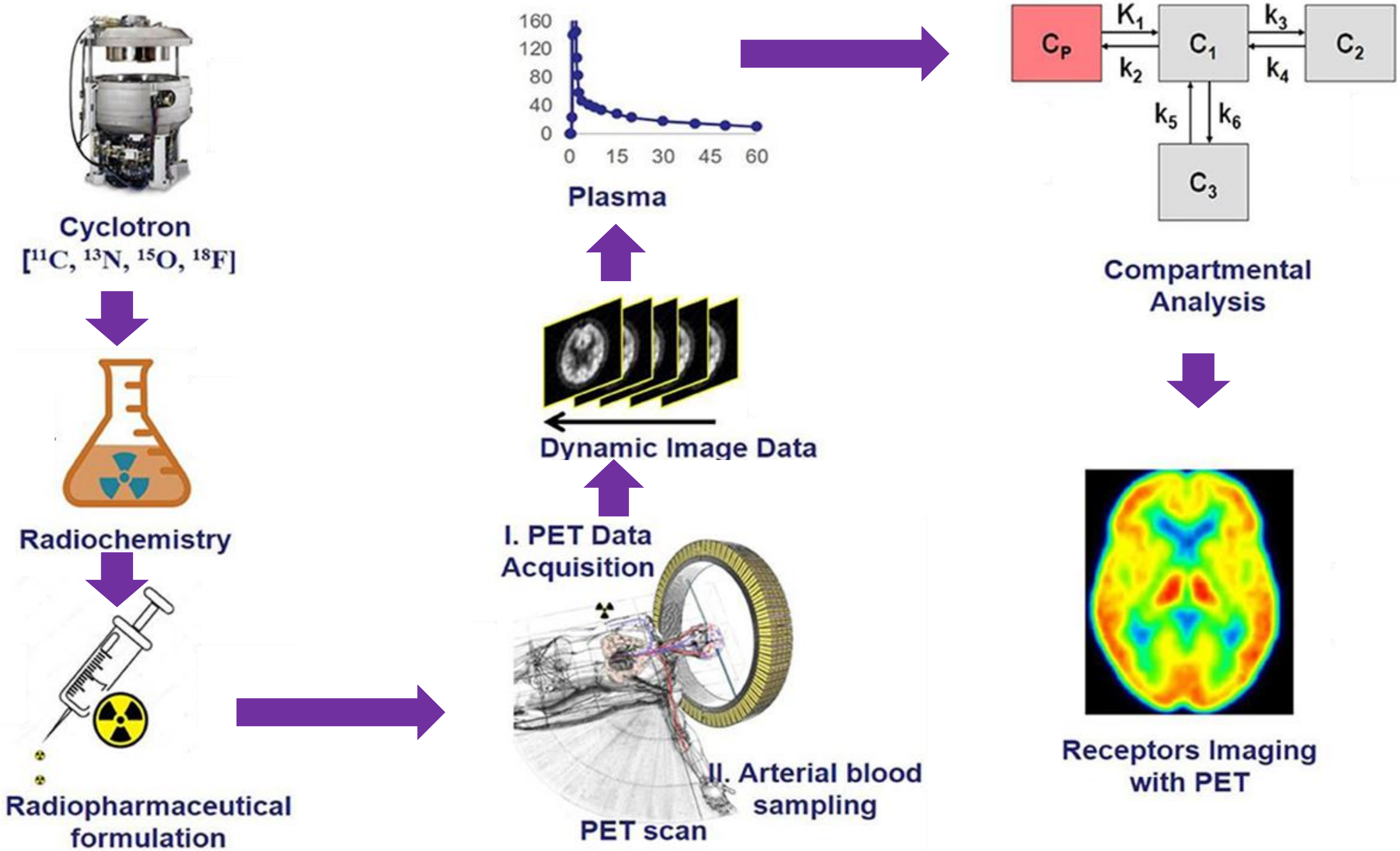
PET (Positron emission tomography)

- **Coincidence detection**: PET detects pairs of gamma photons emitted simultaneously in opposite directions.
- Line of response (LOR) between the detectors is recorded.
- PET images are reconstructed using algorithms such as filtered back projection or iterative methods like Maximum Likelihood Expectation Maximization (MLEM)

$$\rho(x, y, z) = ar\, g \max_{\rho}\left\{\sum_{i=1}^{N} d_i \ln(\rho \cdot R_i) - \rho \cdot R_i\right\}$$

- $\rho(x, y, z)$: reconstructed activity distribution
- d_i: measured data for detector pair *i*
- R_i: system response function for *i*

IMAGE RECONSTRUCTION



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DOSIMETRY

Absorbed dose

$$D = A \cdot E \cdot \phi / m \quad \text{(in grays, Gy)}$$

- A: activity administered
- E: energy of the emitted radiation
- ϕ: fraction of emitted energy absorbed by the tissue
- m: mass of the tissue

Effective dose

$$E = \sum_T w_T D_T$$

- w_T: tissue weighting factor for tissue T
- D_T: absorbed dose to tissue T

depends on the type of radiation and the sensitivity of different tissues

END

To be watch: https://www.youtube.com/watch?v=C01qSc_MNd0&t=1521s

To be watch: https://www.youtube.com/watch?v=_NSyAEi12M0