Series 1 corrections

Exercise 1: Multiple Choice Questions

1. **What are biomedical signals?**
    - A) Signals used for wireless communication
    - B) Signals generated by the human body
    - C) Signals used in mechanical systems
    - D) Signals used for satellite communication
2. **Which of the following is an example of a biomedical signal?**
    - A) Radio waves
    - B) Ultrasound waves
    - C) ECG (Electrocardiogram)
    - D) Seismic waves
3. **Who is considered the pioneer of electrocardiography?**
    - A) Thomas Edison
    - B) Willem Einthoven
    - C) Albert Einstein
    - D) Nikola Tesla
4. **Digital Signal Processing (DSP) is mainly used in biomedical signals for:**
    - A) Cooking food
    - B) Enhancing signal quality and analysis
    - C) Creating computer viruses
    - D) Programming mobile applications
5. **The process of converting an analog biomedical signal to digital is called:**
    - A) Modulation
    - B) Demodulation
    - C) Sampling and Quantization
    - D) Encryption

Exercise 2: True/False Questions

1. False (They can be electrical, mechanical, chemical, etc.)
2. True
3. False (DSP is commonly used for noise removal.)
4. True
5. False (EEG is used to monitor brain activity.)

Exercise 3: Short Answer Questions

1. Biomedical signals provide critical information about physiological conditions, enabling diagnosis, monitoring, and treatment of medical conditions. Examples include ECG for heart monitoring, EEG for brain activity, and EMG for muscle activity.

2. DSP is used in biomedical signal processing for filtering noise, enhancing signal features, analyzing frequency components, and extracting useful information for medical diagnosis.
3. The ECG was developed by Willem Einthoven in the early 1900s. He invented the string galvanometer to record electrical activity of the heart, earning him the Nobel Prize in Physiology or Medicine in 1924. This invention laid the foundation for modern electrocardiography.

Exercise 4:

1. **Identify Noise Frequency:** Analyze the frequency components using the Fast Fourier Transform (FFT).
2. **Choose Filter Type:** Select an appropriate filter (e.g., Low-pass, High-pass, or Band-pass) depending on the noise frequency.
3. **Design the Filter:** Use FIR or IIR filter design techniques.
4. **Apply the Filter:** Convolve the filter with the noisy signal or use digital filtering methods.
5. **Verify Results:** Check the filtered signal using time and frequency domain analysis to ensure noise is removed without distorting the ECG waveform

Exercise 5:

The DFT of a sequence $x[n]$ of length N is given by: $$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{N}kn}$$

$N = 4$ and the sequence is: $x[n] = \{1, 2, 3, 4\}$

$X[k]$ for $k = 0, 1, 2, 3$.

Calculate $X[0]$

$$X[0] = \sum_{n=0}^{3} x[n] \cdot e^{-j\frac{2\pi}{4}\cdot 0\cdot n} = \sum_{n=0}^{3} x[n] = 1 + 2 + 3 + 4 = 10$$

Calculate $X[1]$

$$X[1] = \sum_{n=0}^{3} x[n] \cdot e^{-j\frac{2\pi}{4}\cdot 1\cdot n}$$

$$e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}} = -j$$

$$e^{-j\pi} = -1$$

$$e^{-j\frac{3\pi}{2}} = j$$

$$X[1] = 1 - j \cdot 2 - 3 + j \cdot 4 = 1 - 3 + (-2j + 4j) = -2 + 2j$$

Calculate $X[2]$

$$X[2] = \sum_{n=0}^{3} x[n] \cdot e^{-j\frac{2\pi}{4}\cdot 2\cdot n}$$

$$e^{-j\frac{\pi}{2}\cdot 2} = e^{-j\pi} = -1$$

$$X[2] = 1 - 2 + 3 - 4 = -2$$

Calculate $X[3]$

$$e^{-j\frac{3\pi}{2}} = j$$

$$X[3] = \sum_{n=0}^{3} x[n] \cdot e^{-j\frac{2\pi}{4}\cdot 3\cdot n}$$

$$X[3] = 1 + j \cdot 2 - 3 - j \cdot 4 = 1 - 3 + (2j - 4j) = -2 - 2j$$

The Discrete Fourier Transform of the sequence $x[n] = \{1, 2, 3, 4\}$ is: $X[k] = \{10, -2 + 2j, -2, -2 - 2j\}$

Exercise 6: Sampling Theorem

According to the Nyquist theorem, the maximum frequency component is half of the sampling rate.

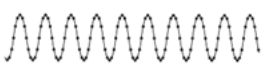
$$f_{max} = \frac{f_s}{2} = \frac{200}{2} = 100 \text{ Hz}$$

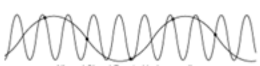
Therefore, the maximum frequency that can be accurately represented is **100 Hz**.

Exercise 7: Aliasing

Aliasing is a common problem in digital signal processing (DSP) that occurs when we sample a continuous signal at a rate lower than twice its highest frequency component. This causes the high-frequency components to appear as lower-frequency ones, distorting the original signal and creating unwanted artifacts

An aliased signal provides a poor representation of the analog signal. Aliasing causes a false lower frequency component to appear in the sampled data of a signal. The following figure shows an adequately sampled signal and an inadequately sampled signal.


Adequate signal


Aliased figure

In the aliased figure, the inadequately sampled signal appears to have a lower frequency than the actual signal (two periods instead of ten periods). Increasing the sampling frequency increases the number of data points acquired in a given time period. Often, a fast sampling frequency provides a better representation of the original signal than a slower sampling frequency.

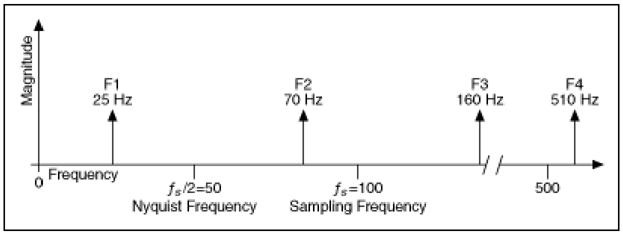
For a given sampling frequency, the maximum frequency you can accurately represent without aliasing is the Nyquist frequency. The Nyquist frequency equals one-half the sampling frequency, as shown by the following equation.

$$f_N = \frac{f_s}{2}$$

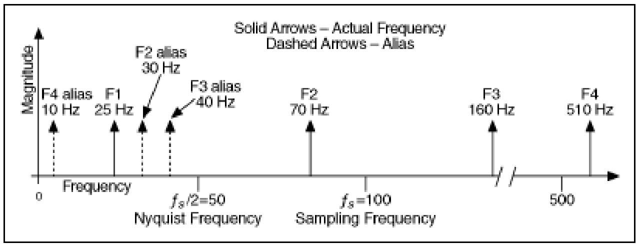
where f_N is the Nyquist frequency and f_s is the sampling frequency.

Signals with frequency components above the Nyquist frequency appear aliased between DC and the Nyquist frequency. In an aliased signal, frequency components actually above the Nyquist frequency appear as frequency components below the Nyquist frequency. For example, a component at frequency $f_N < f_0 < f_s$ appears as the frequency f_s - f_0.

The following two figures show the aliasing phenomenon. The first figure shows the frequencies contained in an input signal acquired at a sampling frequency, f_s, of 100 Hz.



The following figure shows the frequency components and the aliases for the input signal from the previous figure



In the previous figure, frequencies below the Nyquist frequency of $f_s/2$ = 50 Hz are sampled correctly. For example, F1 appears at the correct frequency. Frequencies above the Nyquist frequency appear as aliases. For example, aliases for F2, F3, and F4 appear at 30 Hz, 40 Hz, and 10 Hz, respectively.

The alias frequency equals the absolute value of the difference between the closest integer multiple of the sampling frequency and the input frequency, as shown in the following equation:

AF = |CIMSF - IF|

Where AF is the alias frequency, CIMSF is the closest integer multiple of the sampling frequency, and IF is the input frequency. For example, you can compute the alias frequencies for F2, F3, and F4 from the previous figure with the following equations:

Alias F2 = |100 - 70| = 30 Hz

Alias F3 = |(2)100 - 160| = 40 Hz

Alias F4 = |(5)100 - 510| = 10 Hz

For our exercise, the aliasing occurs when the signal frequency is greater than half the sampling rate.

Nyquist Frequency = $\frac{150}{2} = 75\ \text{Hz}$

Aliased Frequency = $|120 - 150| = 30\ \text{Hz}$

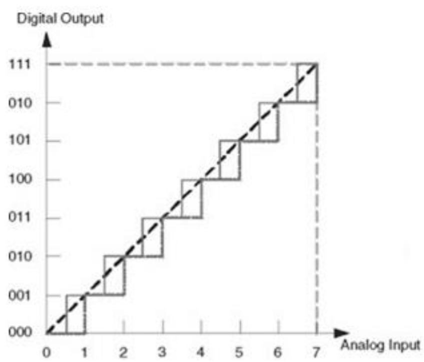
Therefore, the aliased frequency in the sampled signal is **30 Hz**.

Exercise 8: Quantization

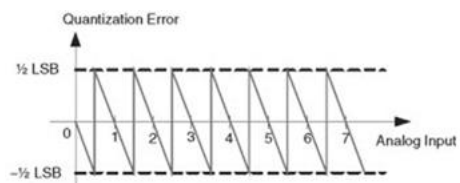
Quantization in signal processing refers to the process of assigning an integer value to the amplitude of a signal at a specific point in time or space

An A/D converter has a finite number of bits (or resolution). As a result, continuous amplitude values get represented or approximated by discrete amplitude levels.

The process of converting continuous into discrete amplitude levels is called quantization. This approximation leads to errors called quantization noise. The input/output characteristic of a 3-bit A/D converter is shown in figure bellow to see how analog voltage values are approximated by discrete voltage levels.



AD convertor input output transfer function



Additive quantization error

A quantization interval depends on the number of quantization or resolution levels, as

We get the step size is by dividing the range by the number of quantization levels.

Number of Levels = $2^{10} = 1024$

Range = $1 - (-1) = 2\ \text{mV}$

Step Size = $\frac{2\ \text{mV}}{1024} = 0.00195\ \text{mV}$

Therefore, the quantization step size is approximately **0.00195 mV**.

Exercise 9: Digital Filtering

The most effective filter for removing a specific frequency is a **Notch Filter**.

Design a Notch Filter with a narrow bandwidth centered at 60 Hz to eliminate the power line interference while preserving the other frequency components of the biomedical signal.

Exercise 10: Discrete Fourier Transform (DFT)

The DFT is calculated using the formula:

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j\frac{2\pi}{N}kn}$$

where $N = 4$

$$X[0] = 1 + 1 + 0 + 0 = 2$$
$$X[1] = 1 + 1 \cdot e^{-j\frac{\pi}{2}} + 0 + 0 = 1 - j$$
$$X[2] = 1 + 1 \cdot e^{-j\pi} + 0 + 0 = 0$$
$$X[3] = 1 + 1 \cdot e^{-j\frac{3\pi}{2}} + 0 + 0 = 1 + j$$

Therefore, the DFT of the sequence is: $X[k] = \{2,1 - j, 0,1 + j\}$

Exercise 11: Convolution in DSP

Convolution is a fundamental operation in digital signal processing that combines two sequences, typically a signal x[n] and an impulse response h[n], to produce an output y[n]. It is used to analyze systems, apply filters, and understand how signals change when passed through a system.

The mathematical definition of convolution for discrete signals is:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

The convolution is calculated such as:

Step-by-Step convolution

Case 1: x[n]={1,2}

Step 1: Write the sequences explicitly

- x[n] is a signal with values at two time instants:x[0]=1,x[1]=2
- h[n] is a filter (or kernel) with values:h[0]=1,h[1]=−1

Step 2: Compute the convolution sum for each n

We calculate y[n] for different values of n.

For n=0: y[0]=x[0]h[0]=(1)(1)=1

For n=1: y[1]=x[0]h[1]+x[1]h[0]=(1)(−1)+(2)(1)=−1+2=1

For n=2n = 2n=2: y[2]=x[1]h[1]=(2)(−1)=−2

$y[n] = x[n] * h[n]$	$y[0] = 1 \cdot 1 = 1$ $y[1] = 1 \cdot (-1) + 2 \cdot 1 = -1 + 2 = 1$ $y[2] = 2 \cdot (-1) = -2$

Therefore, the convolution result is: $y[n] = \{1,1,-2\}$

$$y[0] = x[0] \cdot h[0] = 1 \cdot 1 = 1$$
$$y[1] = x[0] \cdot h[1] + x[1] \cdot h[0] = 1 \cdot (-1) + 2 \cdot 1 = -1 + 2 = 1$$
$$y[2] = x[1] \cdot h[1] + x[2] \cdot h[0] = 2 \cdot (-1) + 3 \cdot 1 = -2 + 3 = 1$$
$$y[3] = x[2] \cdot h[1] = 3 \cdot (-1) = -3$$

Therefore, the convolution result is: $y[n] = \{1,1,1,-3\}$

Exercise 12: FIR Filter Design

FIR filter output is calculated using convolution: $y[n] = x[n] * h[n]$

Step-by-step convolution:

$$y[0] = 1 \cdot 0.2 = 0.2$$
$$y[1] = 1 \cdot 0.5 + 2 \cdot 0.2 = 0.5 + 0.4 = 0.9$$
$$y[2] = 1 \cdot 0.2 + 2 \cdot 0.5 + 1 \cdot 0.2 = 0.2 + 1.0 + 0.2 = 1.4$$
$$y[3] = 2 \cdot 0.2 + 1 \cdot 0.5 + 0 \cdot 0.2 = 0.4 + 0.5 = 0.9$$
$$y[4] = 1 \cdot 0.2 = 0.2$$

Therefore, the output of the FIR filter is: $y[n] = \{0.2,0.9,1.4,0.9,0.2\}$

Exercise 13: FIR Filter Stability

A discrete-time system is stable if its impulse response is absolutely summable:

Calculate the sum: $\sum |h[n]| = |0.3| + |0.3| + |0.4| = 0.3 + 0.3 + 0.4 = 1$

Since the sum is finite, the FIR filter is **stable**.

Exercise 14: Frequency Response of FIR Filter

The frequency response is calculated using the Discrete-Time Fourier Transform (DTFT):

$$H(e^{j\omega}) = \sum_{n=0}^{2} h[n]e^{-j\omega n}$$

$$H(e^{j\omega}) = 1 + e^{-j\omega} + e^{-j2\omega}$$

Note:

$$H(e^{j\omega}) = 1 + cos(\omega) - jsin(\omega) + cos(2\omega) - jsin(2\omega)$$
$$= 1 + 2cos(\omega) + cos(2\omega) - j[sin(\omega) + sin(2\omega)]$$

Magnitude response:

$$H(e^{j\omega}) = \sum_{n=0}^{2} h[n]e^{-j\omega n} \quad = 1 + e^{-j\omega} + e^{-j2\omega}$$

This FIR filter exhibits a low-pass frequency response since the coefficients are all positive and symmetric.