Tutorial 2

Exercise 1: Fundamental Absorption & Indirect Transitions

A semiconductor has an indirect bandgap of 1.1 eV(Silicon). Calculate the minimum photon energy required for an intrinsic transition at room temperature, assuming a phonon of 0.05 eV is available for momentum conservation. For indirect transitions: $h\nu = Eg \pm Ep$ where Ep is the phonon energy (+) for absorption, (−) for emission).

Exercise 2: Exciton & Impurity Absorption

Calculate the binding energy at ground state of an exciton in a semiconductor with a relative permittivity $\epsilon_r = 12$ and a reduced effective mass $m_r^* = 0.05m_0$. We can use the Wannier model for exciton binding energy: $E_{ex} = \frac{13.6eV}{n^2} \times \frac{m_r^*}{m_0} \times \frac{1}{\epsilon_r^2}$

n is the quantum number: is known as the principal quantum number. Just as in a hydrogen atom, it describes the different "energy levels" or orbits that the electron can occupy as it circles the hole.

Exercise 3: Electric Field Effects (Franz-Keldysh & Stark Effects)

Describe the shift in the absorption edge when a high electric field is applied to a bulk semiconductor. Franz-Keldysh Effect (FKE) leads to an absorption tail below the bandgap:

$$\alpha(E,\xi) \propto exp\left(-\frac{4}{3}\frac{\sqrt{2m^*}\left(E_g - E\right)^{3/2}}{eh\xi}\right)$$

$\alpha(E,\xi)$: Absorption coefficient as a function of energy (E) and electric field (ξ).

E_g: Bandgap energy.

m^*: Effective mass of the carrier.

e: Elementary charge.

h: Reduced Planck constant.

Exercise 4. Radiation & Emission-Absorption Relation

Explain the relation between the absorption coefficient α(ν) and the spontaneous emission rate $R_{sp}(\nu)$ using the Van Roosbroeck-Shockley relation**:** Spontaneous emission rate:

$$R_{sp}(\nu) = \frac{8\pi n_r^2 \nu^2}{c^2}\alpha(\nu)\left[exp\left(\frac{h\nu}{k_B T}\right) - 1\right]^{-1}$$

$\alpha(\nu)$ The absorption coefficient at frequency ν

n_r The refractive index of the material.

c The speed of light in vacuum

$h\nu$ The photon energy

k_BT Thermal energy (Boltzmann constant X temperature)

ν^2 is the density of states that comes from the number of possible modes or paths that light can take in 3D crystal.Higher frequencies (shorter wavelengths) have more ways to fit into the crystal structure, so there are more "slots" available for photons to be emitted into.

The fraction $\left[exp\left(\frac{h\nu}{k_BT}\right)-1\right]^{-1}$: This is the Planck distribution (Bose-Einstein statistics), representing the photon density in thermal equilibrium.

Exercise 5:

Complete this table

Process	Governing Energy Condition	Key Feature
Direct Intrinsic		
Indirect Intrinsic		
Exciton		
Impurity		

Exercise 6:

A direct-bandgap semiconductor sample has a bandgap energy Eg=1.43 eV (typical for GaAs). An experimentalist measures the absorption coefficient ($h\nu$) near the bandgap and finds it follows the square-root law for direct transitions: $\alpha(h\nu) = C \cdot (h\nu - Eg)^{1/2}$ for $h\nu > Eg$, where $C = 10^5\ cm^{-1}eV^{-1/2}$

1. Derive the expression for the spontaneous emission spectrum $R_{SP}(h\nu)$ at T=300 K using the Van Roosbroeck-Shockley relation.

$$R_{sp}(h\nu) = \frac{8\pi n_r^2 (h\nu)^2}{c^2 h^3}\alpha(h\nu)\left[\exp\left(\frac{h\nu}{k_BT}\right)-1\right]^{-1}$$

2. Determine the photon energy $(h\nu)_{max}$ at which the emission intensity is maximum. The peak is given by:

$$f(h\nu) = (h\nu - E_g)^{1/2}\exp\left(-\frac{h\nu}{k_BT}\right)$$

3. Calculate the total radiative recombination rate R_0 at equilibrium:

$$R_0 = \int_{E_g}^{\infty} R_{sp}(h\nu)d(h\nu)$$