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The booklet is intended to provide practical help for authors of mathematical papers. It will be useful both as a guide for beginners and as a reference book for experienced writers.

The first part of the booklet provides a useful collection of ready-made sentences and expressions occurring in mathematical papers. The examples are divided into sections according to their use (in introductions, definitions, theorems, proofs, comments, references to the literature, acknowledgments, editorial correspondence and referee's reports). Typical errors are also pointed out.

The second part concerns selected problems of English grammar and usage, most often encountered by mathematical writers. Just as in the first part, an abundance of examples are presented, all of them taken from the actual mathematical texts.

The index enables the reader to find many particular pieces of information scattered throughout the text.

Jerzy Trzeciak, formerly of Polish Scientific Publishers, is now the senior copy editor at the Institute of Mathematics, Polish Academy of Sciences. He is responsible for journals including Studia Mathematica, Fundamenta Mathematicae, Acta Arithmetica and others.

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WRITING MATHEMATICAL PAPERS IN ENGLISH

a practical guide

JERZY TRZECIAK

Copy Editor
Institute of Mathematics
Polish Academy of Sciences

Gdańsk Teachers' Press

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PART A: PHRASES USED IN MATHEMATICAL TEXTS

ABSTRACT AND INTRODUCTION

We prove that in some families of compacta there are no universal elements.
It is also shown that .....
Some relevant counterexamples are indicated.

It is of interest to know whether .....
We are interested in finding .....
It is natural to try to relate ..... to .....
We wish to investigate .....
Our purpose is to .....

This work was intended as an attempt to motivate ⟨at motivating⟩ .....
The aim of this paper is to bring together two areas in which .....

In Section 3 | the third section [*Note*: paragraph $\neq$ section]

- we
  - review some of the standard facts on .....
  - have compiled some basic facts .....
  - summarize without proofs the relevant material on .....
  - give a brief exposition of .....
  - briefly sketch .....
  - set up notation and terminology.
  - discuss ⟨study/treat/examine⟩ the case .....
  - introduce the notion of .....
  - develop the theory of .....
  - will look more closely at .....
  - will be concerned with .....
  - proceed with the study of .....
  - indicate how these techniques may be used to .....
  - extend the results of ..... to .....
  - derive an interesting formula for .....
- it is shown that .....
- some of the recent results are reviewed in a more general setting.
- some applications are indicated.
- our main results are stated and proved.

Section 4
- contains a brief summary ⟨a discussion⟩ of .....
- deals with ⟨discusses⟩ the case .....
- is intended to motivate our investigation of .....
- is devoted to the study of .....
- provides a detailed exposition of .....
- establishes the relation between .....
- presents some preliminaries.

We will
- touch only a few aspects of the theory.
- restrict our attention ⟨the discussion/ourselves⟩ to .....

It is not our purpose to study .....
No attempt has been made here to develop .....
It is possible that ..... but we will not develop this point here.
A more complete theory may be obtained by .....

However,
- this topic exceeds the scope of this paper.
- we will not use this fact in any essential way.

The basic ⟨main⟩
- idea is to apply .....
- geometric ingredient is .....

The crucial fact is that the norm satisfies .....
Our proof involves looking at .....

The proof is
- based on the concept of .....
- similar in spirit to .....
- adapted from .....

This idea goes back at least as far as [7].

We emphasize that .....
It is worth pointing out that .....
The important point to note here is the form of .....
The advantage of using ..... lies in the fact that .....
The estimate we obtain in the course of proof seems to be of independent interest.
Our theorem provides a natural and intrinsic characterization of .....
Our proof makes no appeal to .....
Our viewpoint sheds some new light on .....
Our example demonstrates rather strikingly that .....
The choice of ..... seems to be the best adapted to our theory.

The problem is that .....
The main difficulty in carrying out this construction is that .....
In this case the method of ..... breaks down.
This class is not well adapted to .....
Pointwise convergence presents a more delicate problem.

The results of this paper were announced without proofs in [8].
The detailed proofs will appear in [8] ⟨elsewhere/in a forthcoming publication⟩.
For the proofs we refer the reader to [6].

It is to be expected that .....
One may conjecture that .....
One may ask whether this is still true if .....
One question still unanswered is whether .....
The affirmative solution would allow one to .....
It would be desirable to ..... but we have not been able to do this.
These results are far from being conclusive.
This question is at present far from being solved.

Our method has the disadvantage of not being intrinsic.
The solution falls short of providing an explicit formula.
What is still lacking is an explicit description of .....

As for prerequisites, the reader is expected to be familiar with .....
The first two chapters of ..... constitute sufficient preparation.
No preliminary knowledge of ..... is required.

To facilitate access to the individual topics, the chapters are rendered as self-contained as possible.
For the convenience of the reader we repeat the relevant material from [7] without proofs, thus making our exposition self-contained.

DEFINITION

A set S is *dense* if .....
A set S is called ⟨said to be⟩ *dense* if .....
We call a set *dense* if .....
We call m the *product measure*. [*Note*: The term defined appears *last*.]

The function f is given ⟨defined⟩ by $f =$ .....
Let f be given ⟨defined⟩ by $f =$ .....
We define T to be $AB + CD$.

This map is defined by | requiring f to be constant on .....
| the requirement that f be constant on ..... [Note the infinitive.]
| imposing the following condition: .....

The *length* of a sequence is, by definition, the number of .....
The *length* of T, denoted by $l(T)$, is defined to be .....
By the *length* of T we mean .....

Define ⟨Let/Set⟩ $E = Lf$ | , where | f is .....
| | we have set $f =$ .....
| , f being the solution of .....
| with f satisfying .....

We will consider | the behaviour of the family g defined as follows.
| the height of g (to be defined later) and .....

To measure the growth of g we make the following definition.

In this way we obtain what | we shall call | the *P-system*.
| will be referred to as |
| is known as |

Since ....., | the norm of f is well defined.
| the definition of the norm is unambiguous ⟨makes sense⟩.

It is immaterial which M we choose to define F as long as M contains x.
This product is independent of which member of g we choose to define it.
It is Proposition 8 that makes this definition allowable.

Our definition agrees | with the one given in [7] if u is .....
| with the classical one for .....

Note that | this coincides with our previously introduced terminology if K is convex.
| this is in agreement with [7] for .....

NOTATION

We will denote by Z | the set ..... Write ⟨Let/Set⟩ $f =$ .....
Let us denote by Z | [*Not*: "Denote $f =$ ....."]
Let Z denote |

The closure of A will be denoted by cl$A$.
We will use the symbol ⟨letter⟩ k to denote .....
We write H for the value of .....
We will write the negation of p as $\neg p$.
The notation aRb means that .....
Such cycles are called homologous (written $c \sim c'$).

Here | K | denotes | the map .....
Here and subsequently, | | stands for |
Throughout the proof, |
In the sequel, |
From now on, |

We follow the notation of [8] ⟨used in [8]⟩.
Our notation differs ⟨is slightly different⟩ from that of [8].
Let us introduce the temporary notation Ff for gfg.

With the notation $f =$ ....., | we have .....
With this notation, |
In the notation of [8, Ch. 7] |

If f is real, it is customary to write ..... rather than .....

For simplicity of notation, | we | write f instead of .....
To ⟨simplify/shorten⟩ notation, | | use the same letter f for .....
By abuse of notation, | | continue to write f for .....
For abbreviation, | | let f stand for .....

We abbreviate $Faub$ to b'.
We denote it briefly by F. [*Not*: "shortly"]
We write it F for short ⟨for brevity⟩.
The Radon–Nikodym property (RNP for short) implies that .....
We will write it simply x when no confusion can arise.

It will cause no confusion if we use the same letter to designate a member of A and its restriction to K.

We shall write the above expression as / The above expression may be written as / We can write (4) in the form — $t = \ldots$

The Greek indices label components of sections of E.

Print terminology:

The expression in italics ⟨in italic type⟩, in large type, in bold print;
in parentheses ( ) (= round brackets),
in brackets [ ] (= square brackets),
in braces { } (= curly brackets), in angular brackets ⟨ ⟩;
within the norm signs

Capital letters = upper case letters; small letters = lower case letters;
Gothic ⟨German⟩ letters; script ⟨calligraphic⟩ letters (e.g. $\mathcal{F}$, $\mathcal{G}$);
special Roman letters (e.g. **R**, **N**)

Dot ·, prime ′, asterisk = star *, tilde ˜, bar ¯ [over a symbol], hat ˆ,
vertical stroke ⟨vertical bar⟩ |, slash ⟨diagonal stroke/slant⟩ /,
dash —, sharp #

Dotted line ......, dashed line _ _ _ _ _, wavy line ~~~~~

PROPERTY

the ⟨an⟩ element
- such that ⟨with the property that⟩ .....
  [*Not*: "such an element that"]
- with the following properties: .....
- satisfying $Lf = \ldots$
- with $Nf = 1$ ⟨with coordinates x, y, z⟩
- of norm 1 ⟨of the form .....⟩
- whose norm is .....
- all of whose subsets are .....
- by means of which g can be computed
- for which this is true
- at which g has a local maximum
- described by the equations .....
- given by $Lf = \ldots$
- depending only on ..... ⟨independent of .....⟩
- not in A
- so small that ⟨small enough that⟩ .....
- as above ⟨as in the previous theorem⟩
- so obtained
- occurring in the cone condition
  [Note the double "r".]
- guaranteed by the assumption .....

the ⟨an⟩ element
- we have just defined
- we wish to study ⟨we used in Chapter 7⟩
- to be defined later [= which will be defined]
- in question
- under study ⟨consideration⟩

....., the constant C being independent of ..... [= where C is .....]
....., the supremum being taken over all cubes .....
....., the limit being taken in L.

....., where C
- is so chosen that .....
- is to be chosen later.
- is a suitable constant.
- is a conveniently chosen element of .....
- involves the derivatives of .....
- ranges over all subsets of .....
- may be made arbitrarily small by .....

The operators A_i
- have ⟨share⟩ many of the properties of .....
- have still better smoothness properties.
- lack ⟨fail to have⟩ the smoothness properties of .....
- still have norm 1.
- are
  - not merely symmetric but actually self-adjoint.
  - not necessarily monotone.
  - both symmetric and positive-definite.
  - not continuous, nor do they satisfy (2).
    [Note the inverse word order after "nor".]
  - neither symmetric nor positive-definite.
  - only nonnegative rather than strictly positive, as one may have expected.
  - any self-adjoint operators, possibly even unbounded.
  - still ⟨no longer⟩ self-adjoint.
  - not too far from being self-adjoint.

The
- preceding theorem
- indicated set
- above-mentioned group
- resulting region
- required ⟨desired⟩ element

[But adjectival clauses with prepositions come *after* a noun, e.g. "the group defined in Section 1".]

Both X and Y are finite.
Neither X nor Y is finite.
X and Y are countable, but neither is finite.
Neither of them is finite. [*Note*: "Neither" refers to *two* alternatives.]
None of the functions F_i is finite.

X is not finite; nor ⟨neither⟩ is Y.

X is not finite, nor is Y countable. [Note the inversion.]

X is empty | ; so is Y.
| , but Y is not.

X belongs to Y | ; so does Z.
| , but Z does not.

ASSUMPTION, CONDITION, CONVENTION

We will make ⟨need⟩ the following assumptions: .....
From now on we make the assumption: .....
The following assumption will be needed throughout the paper.
Our basic assumption is the following.
Unless otherwise stated ⟨Until further notice⟩ we assume that .....
In the remainder of this section we assume ⟨require⟩ g to be .....
In order to get asymptotic results, it is necessary to put some restrictions on f.
We shall make two standing assumptions on the maps under consideration.

It is required ⟨assumed⟩ that .....
The requirement on g is that .....

....., where g | is subject to the condition $Lg = 0$.
| satisfies the condition $Lg = 0$.
| is merely required to be positive.

Let us orient M by | the requirement that g be positive. [Note the infinitive.]
| requiring g to be .....
| imposing the condition: .....

(4) holds | for ⟨provided/whenever/only in case⟩ $p \neq 1$.
| unless $p = 1$.
| under | the condition ⟨hypothesis⟩ that .....
| | the more general assumption that .....
| | some further restrictions on .....
| | additional ⟨weaker⟩ assumptions.

F | satisfies ⟨fails to satisfy⟩ the assumptions of .....
| has the desired ⟨asserted⟩ properties.
| provides the desired diffeomorphism.
| still satisfies ⟨need not satisfy⟩ the requirement that .....
| meets this condition.
| does not necessarily have this property.
| satisfies all the other conditions for membership of X.

There is no loss of generality in assuming .....
Without loss ⟨restriction⟩ of generality we can assume .....

This involves no loss of generality.

We can certainly assume that ..... | , since otherwise .....
| , for ..... [= because]
| , for if not, we replace .....
| . Indeed, ...

Neither the hypothesis nor the conclusion is affected if we replace .....
By choosing $b = a$ we may actually assume that .....
If $f = 1$, which we may assume, then .....
For simplicity ⟨convenience⟩ we ignore the dependence of F on g. [E.g. in notation]
It is convenient to choose .....
We can assume, by decreasing k if necessary, that .....

F meets S transversally, say at $F(0)$.
There exists a minimal element, say n, of F.
G acts on H as a multiple (say n) of V.
For definiteness ⟨To be specific⟩, consider .....

This condition | is not particularly restrictive.
| is surprisingly mild.
| admits ⟨rules out/excludes⟩ elements of .....
| is essential to the proof.
| cannot be weakened ⟨relaxed/improved/omitted/dropped⟩.

The theorem is true if "open" is deleted from the hypotheses.
The assumption ..... is superfluous ⟨redundant/unnecessarily restrictive⟩.
We will now show how to dispense with the assumption on .....
Our lemma does not involve any assumptions about curvature.

We have been working under the assumption that .....
Now suppose that this is no longer so.
To study the general case, take .....
For the general case, set .....

The map f will be viewed ⟨regarded/thought of⟩ as | a functor .....
| realizing .....

From now on we | think of L as being constant.
| regard f as a map from .....
| tacitly assume that .....

It is understood that $r \neq 1$.
We adopt ⟨adhere to⟩ the convention that $0/0=0$.

THEOREM: GENERAL REMARKS

This theorem	is	an extension ⟨a fairly straightforward generalization/a sharpened version/a refinement⟩ of .....
		an analogue of .....
		a reformulation ⟨restatement⟩ of ..... in terms of .....
		analogous to .....
		a partial converse of .....
		an answer to a question raised by .....
	deals with	
	ensures the existence of	
	expresses the equivalence of	
	provides a criterion for	
	yields information about	
	makes it legitimate to apply	

The theorem states ⟨asserts/shows⟩ that .....
Roughly ⟨Loosely⟩ speaking, the formula says that .....

When f is open, (3.7) just amounts	to saying that .....
	to the fact that .....

Here is another way of stating (c): .....
Another way of stating (c) is to say: .....
An equivalent formulation of (c) is: .....
Theorems 2 and 3 may be summarized by saying that .....
Assertion (ii) is nothing but the statement that .....
Geometrically speaking, the hypothesis is that .....: part of the conclusion is that .....

The interest	of the lemma is	in the assertion .....
The principal significance		that it allows one to .....
The point		

The theorem gains in interest if we realize that .....

The theorem	is still true	if	we drop the assumption .....
	still holds		it is only assumed that .....

If we take $f =$	we recover	the standard lemma .....
Replacing f by $-f$,		[7, Theorem 5].

This specializes to the result of [7] if $f = g$.

This result will	be needed in	Section 8.
	prove extremely useful in	
	not be needed until	

THEOREM: INTRODUCTORY PHRASE

We have thus proved	We can now	rephrase Theorem 8 as follows.
Summarizing, we have		state the analogue of .....
		formulate our main results.

We are thus led to the following strengthening of Theorem 6: .....
The remainder of this section will be devoted to the proof of .....

The continuity of A is established by our next theorem.
The following result may be proved in much the same way as Theorem 6.
Here are some elementary properties of these concepts.
Let us mention two important consequences of the theorem.
We begin with a general result on such operators.

[*Note*: Sentences of the type "We now have the following lemma", carrying no information, can in general be cancelled.]

THEOREM: FORMULATION

If ..... and if ....., then .....

Let M be	Suppose that	Then .....,	provided $m \neq 1$.
	Assume that		unless $m = 1$.
	Write		with g a constant satisfying .....

Furthermore ⟨Moreover⟩, ...
In fact, ... [= To be more precise]
Accordingly, ... [= Thus]

Given any $f \neq 1$ suppose that ..... Then .....

Let P satisfy	the hypotheses of	Then .....
	the above assumptions.	
	$N(P) = 1$.	

Let assumptions 1–5 hold. Then .....
Under the above assumptions, ...
Under the same hypotheses, ...
Under the conditions stated above, ...
Under the assumptions of Theorem 2 with "convergent" replaced by "weakly convergent", ...
Under the hypotheses of Theorem 5, if moreover .....

Equality holds in (8) if and only if .....
The following conditions are equivalent: .....

[*Note*: Expressions like "the following inequality holds" can in general be dropped.]

PROOF: BEGINNING

We / Let us | first |
- prove ⟨show/recall/observe⟩ that .....
- prove a reduced form of the theorem.
- outline ⟨give the main ideas of⟩ the proof.
- examine Bf.

But $A = B$. |
- To see ⟨prove⟩ this, let $f =$ .....
- We prove this as follows.
- This is proved by writing $g =$ .....

We first compute If. |
- To this end, consider .....
  [= For this purpose; *not*: "To this aim"]
- To do this, take .....
- For this purpose, we set .....

To deduce (3) from (2), take .....
We claim that ..... Indeed, ...
We begin by proving ..... ⟨by recalling the notion of .....⟩

Our proof starts with the observation that .....
The procedure is to find .....
The proof consists in the construction of .....

The proof is |
- straightforward ⟨quite involved⟩.
- by induction on n.
- left to the reader.
- based on the following observation.

The main ⟨basic⟩ idea of the proof is to take .....

The proof |
- falls naturally into three parts.
- will be divided into 3 steps.

We have divided the proof into a sequence of lemmas.

Suppose |
- the assertion of the lemma is false.
- , contrary to our claim, that .....

Conversely ⟨To obtain a contradiction⟩, / On the contrary, | suppose that .....

Suppose the lemma were false. Then we could find .....

If |
- there existed an x .....,
- x were not in B,
- it were true that .....,

| we would have ..... / there would be .....

Assume the formula holds for the degree k; we will prove it for $k+1$.
Assuming (5) to hold for k, we will prove it for $k+1$.

We give the proof only for the case $n = 3$; the other cases are left to the reader.
We only give the main ideas of the proof.

PROOF: ARGUMENTS

By |
- definition, ...
- the definition of .....
- assumption, ...
- the compactness of .....
- Taylor's formula, ...
- a similar argument, ...
- the above, ...
- the lemma below, ...
- continuity, ...

But $f = g$ |
- , which follows from .....
- as was described ⟨shown/mentioned/noted⟩ in .....

Theorem 4 now |
- shows that .....
- yields ⟨gives/implies⟩ $f =$ .....
- leads to $f =$ .....

Since f is compact, |
- $Lf = 0$. [*Not*: "Since ....., then ....."]
- we have $Lf = 0$.
- it follows that $Lf = 0$.
- we see ⟨conclude⟩ that $Lf = 0$.

But $Lf = 0$ since f is compact.
We have $Lf = 0$, because ..... [+ a longer explanation]
We must have $Lf = 0$, for otherwise we can replace .....
As f is compact we have $Lf = 0$.
Therefore $Lf = 0$ by Theorem 6.
That $Lf = 0$ follows from Theorem 6.

From | this / (5) / what has already been proved, |
- we conclude ⟨deduce/see⟩ that .....
- we have ⟨obtain⟩ $M = N$.
  [*Note*: without "that"]
- it follows that .....
- it may be concluded that .....

According to ⟨On account of⟩ the above remark, we have $M=N$.

It follows that / Hence ⟨Thus/Consequently,/Therefore⟩ | $M=N$.

[hence = from this; thus = in this way; therefore = for this reason; it follows that = from the above it follows that]

This gives $M=N$.
We thus get $M=N$.
The result is $M=N$.
(3) now becomes $M=N$.
This clearly forces $M=N$.

F is compact, |
- and so $M=N$.
- and consequently $M=N$.
- and, in consequence, $M=N$.
- and hence bounded.
- which gives ⟨implies/yields⟩ $M=N$.
  [*Not*: "what gives"]

$F=G=H$, |
- the last equality being a consequence of Theorem 7.
- which is due to the fact that .....

Since ....., (2) shows that ....., by (4).
We conclude from (5) that ....., hence that ....., and finally that .....

The equality $f=g$, which is part of the conclusion of Theorem 7, implies that .....

As in the proof of Theorem 8, equation (4) gives .....
Analysis similar to that in the proof of Theorem 5 shows that ..... [*Not*: "similar as in"]
A passage to the limit similar to the above implies that .....
Similarly ⟨Likewise⟩, ...

Similar arguments apply The same reasoning applies	to the case .....

The same conclusion can be drawn for .....
This follows by the same method as in .....
The term Tf can be handled in much the same way, the only difference being in the analysis of .....
In the same manner we can see that .....
The rest of the proof runs as before.
We now apply this argument again, with I replaced by J, to obtain .....

PROOF: CONSECUTIVE STEPS

Consider ..... Choose ..... Fix	Define Let Set	$f=$	Let us	evaluate ..... compute ..... apply the formula to ..... suppose for the moment ..... regard s as fixed and .....

[*Note*: The imperative mood is used when you *order* the reader to do something, so you should not write e.g. "Give an example of ....." if you mean "*We* give an example of ....."]

Adding g to the left-hand side Subtracting (3) from (5) Writing ⟨Taking⟩ $h=Hf$ Substituting (4) into (6) Combining (3) with (6) Combining these [E.g. these inequalities] Replacing (2) by (3) Letting $n\to\infty$ Applying (5) Interchanging f and g	yields ⟨gives⟩ $h=$ ..... we obtain ⟨get/have⟩ $f=g$ [*Note*: without "that"] we conclude ⟨deduce/see⟩ that ..... we can assert that ..... we can rewrite (5) as .....

[*Note*: The ing-form is either the subject of a sentence ("Adding ..... gives"), or requires the subject "we" ("Adding ..... we obtain"); so do *not* write e.g. "Adding ..... the proof is complete."]

We continue in this fashion obtaining ⟨to obtain⟩ $f=$ .....
We may now integrate k times to conclude that .....

Repeated application of Lemma 6 enables us to write .....
We now proceed by induction.
We can now proceed analogously to the proof of .....

We next	claim ⟨show/prove that⟩ ..... sharpen these results and prove that .....

Our next	claim is that ..... goal is to determine the number of ..... objective is to evaluate the integral I. concern will be the behaviour of .....

We now turn to the case $f\neq 1$.
We are now in a position to show ..... [= We are able to]
We proceed to show that .....
The task is now to find .....
Having disposed of this preliminary step, we can now return to .....

We wish to arrange that f be as smooth as possible. [Note the infinitive.]
We are thus looking for the family .....
We have to construct .....

In order to get this inequality, it	will be necessary to ..... is convenient to .....

To deal with If, To estimate the other term, For the general case,	we note that .....

PROOF: "IT IS SUFFICIENT TO ....."

It	suffices is sufficient	to	show ⟨prove⟩ that ..... make the following observation. use (4) together with the observation that .....

We need only consider 3 cases: .....
We only need to show that .....

It remains to prove that ..... ⟨to exclude the case when .....⟩
What is left is to show that .....
We are reduced to proving (4) for .....
We are left with the task of determining .....
The only point remaining concerns the behaviour of .....
The proof is completed by showing that .....
We shall have established the lemma if we prove the following: .....
If we prove that ....., the assertion follows.
The statement $O(g)=1$ will be proved once we prove the lemma below.

PROOF: "IT IS EASILY SEEN THAT ....."

It is | clear ⟨evident/immediate/obvious⟩ that .....
| easily seen that .....
| easy to check that .....
| a simple matter to .....

We see ⟨check⟩ at once that .....
F is easily seen ⟨checked⟩ to be smooth.

....., which is clear from (3).
....., as is easy to check.

It follows easily ⟨immediately⟩ that .....
Of course ⟨Clearly/Obviously⟩, ...
The proof is straightforward ⟨standard/immediate⟩.

An easy computation ⟨A trivial verification⟩ shows that .....
(2) makes it obvious that ..... [= By (2) it is obvious that]

The factor Gf poses no problem because G is .....

PROOF: CONCLUSION AND REMARKS

....., which [*Not*: "what"] / This | proves the theorem.
| completes the proof.
| establishes the formula.
| is the desired conclusion.
| is our claim ⟨assertion⟩. [*Not*: "thesis"]
| gives (4) when substituted in (5) ⟨combined with (5)⟩.

....., and | the proof is complete.
| this is precisely the assertion of the lemma.
| the lemma follows.
| (3) is proved.
| $f=g$ as claimed ⟨required⟩.

This contradicts our assumption ⟨the fact that .....⟩.
....., contrary to (3).
....., which is impossible. [*Not*: "what is"]
....., which contradicts the maximality of .....
....., a contradiction.

The proof for G is similar.
G may be handled in much the same way.
Similar considerations apply to G.

The same proof | works ⟨remains valid⟩ for .....
| still goes ⟨fails⟩ when we drop the assumption .....

The method of proof carries over to domains .....

The proof above gives more, namely f is .....
A slight change in the proof actually shows that .....

Note that we have actually proved that .....
[= We have proved more, namely that .....]

We have used | only the fact that .....
| the existence of only the right-hand derivative.

For $f=1$ | it is no longer true that .....
| the argument breaks down.

The proof strongly depended on the assumption that .....

Note that we did not really have to use .....; we could have applied .....

For more details we refer the reader to [7].
The details are left to the reader.
We leave it to the reader to verify that ..... [*Note*: "it" necessary]
This finishes the proof, the detailed verification of (4) being left to the reader.

REFERENCES TO THE LITERATURE

(see for instance [7, Th. 1]) (see [7] and the references given there)

(see [Ka2] for | more details)
| the definition of .....)
| the complete bibliography)

The best general reference here / The standard work on ..... / The classical work here | is .....

This | was proved by Lax [8].
| can be found in Lax [7, Ch. 2].

This construction | is due to Strang [8].
| goes back | to the work of .....
| | as far as [8].
| was motivated by [7].
| generalizes that of [7].
| follows [7].
| is adapted from [7] ⟨appears in [7]⟩.
| has previously been used by Lax [7].

For | a recent account of the theory
| a treatment of a more general case
| a fuller ⟨thorough⟩ treatment
| a deeper discussion of .....
| direct constructions along more classical lines
| yet another method
| we refer the reader to [7].

We introduce the notion of ....., following Kato [7].
We follow [Ka] in assuming that .....

The main results of this paper were announced in [7].
Similar results have been obtained independently by Lax and are to be published in [7].

ACKNOWLEDGMENTS

The author | wishes to express his thanks ⟨gratitude⟩ to .....
| is greatly indebted to .....

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HOW TO SHORTEN THE PAPER

General rules:

1. Remember: you are writing for an expert. Cross out all that is trivial or routine.
2. Avoid repetition: do not repeat the assumptions of a theorem at the beginning of its proof, or a complicated conclusion at the end of the proof. Do not repeat the assumptions of a previous theorem in the statement of a next one (instead, write e.g. "Under the hypotheses of Theorem 1 with f replaced by g, ..."). Do not repeat the same formula—use a label instead.
3. Check all formulas: is each of them necessary?

Phrases you can cross out:

We denote by **R** the set of all real numbers.
We have the following lemma.
The following lemma will be useful.
..... the following inequality is satisfied:

Phrases you can shorten (see also p. 38):

Let ε be an arbitrary but fixed positive number ⇝ Fix $\varepsilon > 0$
Let us fix arbitrarily $x \in X$ ⇝ Fix $x \in X$
Let us first observe that ⇝ First observe that
We will first compute ⇝ We first compute

Hence we have $x=1$ ⇝ Hence $x=1$
Hence it follows that $x=1$ ⇝ Hence $x=1$

Taking into account (4) ⇝ By (4)
By virtue of (4) ⇝ By (4)
By relation (4) ⇝ By (4)

In the interval $[0,1]$ ⇝ In $[0,1]$
There exists a function $f \in C(X)$ ⇝ There exists $f \in C(X)$
For every point $p \in M$ ⇝ For every $p \in M$
F is defined by the formula $F(x)=.....$ ⇝ F is defined by $F(x)=.....$

Theorem 2 and Theorem 5 ⇝ Theorems 2 and 5
This follows from (4), (5), (6) and (7) ⇝ This follows from (4)–(7)
For details see [3], [4] and [5] ⇝ For details see [3]–[5]

The derivative with respect to t ⇝ The t-derivative
A function of class C^2 ⇝ A C^2 function

For arbitrary x ⇝ For all x ⟨For every x⟩
In the case $n=5$ ⇝ For $n=5$
This leads to a contradiction with the maximality of f ⇝ ....., contrary to the maximality of f
Applying Lemma 1 we conclude that ⇝ Lemma 1 shows that
....., which completes the proof ⇝ ..... ∎

EDITORIAL CORRESPONDENCE

I would like to submit | the enclosed manuscript "....."
I am submitting | for publication in Studia Mathematica.

I have also included a reprint of my article ..... for the convenience of the referee.

I wish to withdraw my paper ..... as I intend to make a major revision of it.
I regret any inconvenience this may have caused you.

I am very pleased that the paper will appear in Fundamenta.
Thank you very much for accepting my paper for publication in .....

Please find enclosed two copies of the revised version.
As the referee suggested, I inserted a reference to the theorem of .....
We have followed the referee's suggestions.
I have complied with almost all suggestions of the referee.

REFEREE'S REPORT

The author proves the interesting result that .....
The proof is short and simple, and the article well written.
The results presented are original.

The paper is a good piece of work on a subject that attracts considerable attention.

I am pleased to It is a pleasure to I strongly	recommend it for publication in Studia Mathematica.

The only remark I wish to make is that condition B should be formulated more carefully.

A few minor typographical errors are listed below.

I have indicated various corrections on the manuscript.

The results obtained are not particularly surprising and will be of limited interest.

The results are	correct but only moderately interesting. rather easy modifications of known facts.

The example is worthwhile but not of sufficient interest for a research article.

The English of the paper needs a thorough revision.

The paper does not meet the standards of your journal.

Theorem 2 is false	as stated. in this generality.

Lemma 2 is known (see .....)
Accordingly, I recommend that the paper be rejected.

PART B: SELECTED PROBLEMS OF ENGLISH GRAMMAR

INDEFINITE ARTICLE (a, an, —)

Note: You use "a" or "an" depending on *pronunciation* and not spelling, e.g. a unit, an x.

1. **Instead of the number "one"**:

The four centres lie in **a** plane.
A chapter will be devoted to the study of expanding maps.
For this, we introduce **an** auxiliary variable z.

2. **Meaning "member of a class of objects", "some", "one of"**:

Then D becomes **a** locally convex space with dual space D'.
The right-hand side of (4) is then **a** bounded function.
This is easily seen to be **an** equivalence relation.
Theorem 7 has been extended to **a** class of boundary value problems.
The transitivity is **a** consequence of the fact that .....
Let us now state **a** corollary of Lebesgue's theorem for .....
After **a** change of variable in the integral we get .....
We thus obtain the estimate ..... with **a** constant C.

in the plural:

The existence of partitions of unity may be proved by .....
The definition of distributions implies that .....
....., with suitable constants.
....., where G and F are differential operators.

3. **In definitions of classes of objects (i.e. when there are many objects with the given property)**:

A fundamental solution is a function satisfying .....
We call C **a** module of ellipticity.
A classical example of **a** constant C such that .....
We wish to find **a** solution of (6) which is of the form .....

in the plural:

The elements of D are often called test functions.

the set of	points with distance 1 from K all functions with compact support

The integral may be approximated by sums of the form .....
Taking in (4) functions v which vanish in U we obtain .....
Let f and g be functions such that .....

4. **In the plural—when you are referring to each element of a class:**

Direct sums exist in the category of abelian groups.
In particular, closed sets are Borel sets.
Borel measurable functions are often called Borel mappings.
This makes it possible to apply H_2-results to functions in any H_p.

If you are referring to all elements of a class, you use "the":

The real measures form a subclass of **the** complex ones.

5. **In front of an adjective which is intended to mean "having this particular quality":**

This map extends to all of M in **an** obvious fashion.
A remarkable feature of the solution should be stressed.

Section 1 | gives **a** condensed exposition of .....
| describes in **a** unified manner the recent results .....

A simple computation gives .....
Combining (2) and (3) we obtain, with **a** new constant C, ...
A more general theory must be sought to account for these irregularities.
The equation (3) has **a** unique solution g for every f.
But: (3) has **the** unique solution $g = ABf$.

DEFINITE ARTICLE (the)

1. **Meaning "mentioned earlier", "that":**

Let $A \subset X$. If $aB = 0$ for every B intersecting **the** set A, then .....
Define $\exp x = \Sigma x^i/i!$. **The** series can easily be shown to converge.

2. **In front of a noun (possibly preceded by an adjective) referring to a single, uniquely determined object (e.g. in definitions):**

Let f be **the** linear form | $g \to (g, F)$.
| defined by (2). [If there is only one.]

$u = 1$ in **the** compact set K of all points at distance 1 from L.
We denote by $B(X)$ **the** Banach space of all linear operators in X.
....., under **the** usual boundary conditions.
....., with **the** natural definitions of addition and multiplication.
Using **the** standard inner product we may identify .....

3. **In the construction: the + property (or another characteristic) + of + object:**

The continuity of f follows from .....
The existence of test functions is not evident.
There is a fixed compact set containing **the supports of** all the f^j.
Then x is **the centre of** an open ball U.
The intersection of a decreasing family of such sets is convex.

But: Every nonempty open set in $\mathbf{R}^k$ is **a union of** disjoint boxes.
[If you wish to stress that it is some union of not too well specified objects.]

4. **In front of a cardinal number if it embraces all objects considered:**

The two groups have been shown to have the same number of generators. [Two groups only were mentioned.]
Each of **the** three products on the right of (4) satisfies .....
[There are exactly 3 products there.]

5. **In front of an ordinal number:**

The first Poisson integral in (4) converges to g.
The second statement follows immediately from **the first**.

6. **In front of surnames used attributively:**

the Dirichlet problem	*But*:	Taylor's formula
the Taylor expansion		[without "the"]
the Gauss theorem		**a** Banach space

7. **In front of a noun in the plural if you are referring to a class of objects as a whole, and not to particular members of the class:**

The real measures form a subclass of **the** complex ones.
This class includes **the** Helson sets.

ARTICLE OMISSION

1. **In front of nouns referring to activities:**

Application of Definition 5.9 gives (45).
Repeated application ⟨use⟩ of (4.8) shows that .....
The last formula can be derived by direct consideration of .....
A is the smallest possible extension in which differentiation is always possible.
Using integration by parts we obtain .....
If we apply induction to (4), we get .....
Addition of (3) and (4) gives .....
This reduces the solution to division by Px.
Comparison of (5) and (6) shows that .....

[*Note*: In constructions with "of" you can also use "the".]

2. **In front of nouns referring to properties if you mention no particular object:**

In question of uniqueness one usually has to consider .....
By continuity, (2) also holds when $f = 1$.
By duality we easily obtain the following theorem.
Here we do not require translation invariance.

3. After certain expressions with "of":

a **type of** convergence
a **problem of** uniqueness
the **condition of** ellipticity
the **hypothesis of** positivity
the **method of** proof
the **point of** increase

4. In front of numbered objects:

It follows from **Theorem 7** that .....
Section 4 gives a concise presentation of .....
Property (iii) is called the triangle inequality.
This has been proved in **part (a)** of the proof.

But: the set of solutions of **the** form (4.7)
To prove **the** estimate (5.3) we first extend .....
We thus obtain **the** inequality (3). [*Or*: inequality (3)]
The asymptotic formula (3.6) follows from .....
Since **the** region (2.9) is in U, we have .....

5. To avoid repetition:

the order and symbol of a distribution
the associativity and commutativity of A
the direct sum and direct product
the inner and outer factors of f [Note the plural.]

But: **a** deficit or **an** excess

6. In front of surnames in the possessive:

Minkowski's inequality, *but*: **the** Minkowski inequality
Fefferman and Stein's famous theorem,
more usual: the famous Fefferman–Stein theorem

7. In some expressions describing a noun, especially after "with" and "of":

an algebra **with** unit e; an operator **with** domain H^2; a solution **with** vanishing Cauchy data; a cube **with** sides parallel to the axes; a domain **with** smooth boundary; an equation **with** constant coefficients; a function **with** compact support; random variables **with** zero expectation
the equation **of** motion; the velocity **of** propagation;
an element **of** finite order; a solution **of** polynomial growth;
a ball **of** radius 1; a function **of** norm p

But: elements of **the** form $f = \dots$
Let B be a Banach space with **a** weak symplectic form w.
Two random variables with **a** common distribution.

8. After "to have":

F has | finite norm.
| compact support.

But: F has | **a** finite norm not exceeding 1.
| **a** compact support contained in I.

F has | rank 2.
| cardinality c.
| absolute value 1.
| determinant zero.

But: F has | **a** zero of order at least 2 at the origin.
| **a** density g.
[Unless g has appeared earlier; then: F has density g.]

9. In expressions with "as":

Any random variable can be taken **as** coordinate variable on Y.
Here t is interpreted **as** area or volume.
We show that G is a group with composition **as** group operation.

But: G is well defined as **the** integral of f over U.

10. In front of the name of a mathematical discipline:

This idea comes from game theory ⟨homological algebra⟩.

But: in **the theory of** distributions

11. Other examples:

We can assume that G is **in diagonal form**.
Then A is deformed into B by pushing it **at constant speed** along the integral curves of X.
G is now viewed as a set, **without group structure**.

INFINITIVE

1. Indicating aim or intention:

To prove the theorem, we first let .....

We now apply (5) | **to study** the group of .....
| **to derive** the following theorem.
| **to obtain** an x with norm not exceeding 1.

Here are some examples **to show** how .....

2. In constructions with "too" and "enough":

This method is **too** complicated **to** be used here.
This case is important **enough to** be stated separately.

3. Indicating that one action leads to another one:

We now apply Theorem 7 **to get** $Nf = 0$. [= ..... and we get $Nf = 0$]
Insert (2) into (3) **to find** that .....

4. In constructions like "we may assume M to be .....":

We may **assume** M **to be** compact.
We **define** K **to be** the section of H over S.
If we **take** the contour G **to lie** in U, then .....
We **extend** f **to be** homogeneous of degree 1.
The class A is defined by **requiring** all the functions f **to satisfy** .....
Partially order P by **declaring** $X < Y$ **to mean** that .....

5. In constructions like "M is assumed to be .....":

M
- is **assumed** ⟨expected/found/considered/taken/claimed⟩ **to be** open.
- will be **chosen to contain** 0.
- can be **taken to be** a constant.
- can easily be **shown to have** ..... [*Note*: "easily" after "can"]
- is also **found to be** of class S.

This investigation is **likely to produce** good results.
[= It is very probable it will]
The close agreement of the six elements is **unlikely to be** a coincidence. [= is probably not]

6. In the structure "for this to happen":

For this to happen, F must be compact.
[= In order that this happens]
For the last **estimate to hold**, it is enough to assume .....
Then **for such a map to exist**, we must have .....

7. As the subject of a sentence:

To see that this is not a symbol **is** fairly easy.
[*Or*: It is fairly easy to see that .....]
To choose a point at random in the interval $[0, 1]$ **is** a conceptual experiment with an obvious intuitive meaning.
To say that u is maximal **means** simply that .....

After expressions with "it":

It is necessary ⟨useful/very important⟩ **to consider** .....
It makes sense to speak of .....
It is therefore **of interest to look** at .....

8. After "be":

Our goal ⟨method/approach/procedure/objective/aim⟩ **is to find** .....
The problem ⟨difficulty⟩ here **is to construct** .....

9. With nouns and with superlatives, in the place of a relative clause:

The theorem **to be proved** is the following. [= which will be proved]
This will be proved by the method **to be described** in Section 6.
For other reasons, **to be discussed** in Chapter 4, we have to .....
He was **the first to propose** a complete theory of .....
They appear to be **the first to have suggested** the now accepted interpretation of .....

10. After certain verbs:

These properties **led** him **to suggest** that .....
They **believe to have discovered** .....
Lax **claims to have obtained** a formula for .....
This map **turns out to satisfy** .....

At first glance M **appears to differ** from N in two major ways: .....
A more sophisticated argument **enables** one **to prove** that .....
[*Note*: "enable" requires "one", "us" etc.]
He **proposed to study** that problem. [*Or*: He proposed studying .....]
We **make** G **act** trivially on V.
Let f **satisfy** (2). [*Not*: "satisfies"]
We **need to consider** the following three cases.
We **need not consider** this case separately.
["need to" in affirmative clauses, without "to" in negative clauses; also note: "we only need to consider", but: "we need only consider"]

ING-FORM

1. As the subject of a sentence (note the absence of "the"):

Repeating the previous argument and **using** (3) **leads** to .....
Since **taking** symbols **commutes** with lifting, A is .....
Combining Proposition 5 and Theorem 7 **gives** .....

2. After prepositions:

After making a linear transformation, we may assume that .....
In passing from (2) to (3) we have ignored the factor n.
In deriving (4) we have made use of .....
On substituting (2) into (3) we obtain .....
Before making some other estimates, we prove .....
Z enters X **without meeting** $x = 0$.
Instead of using the Fourier method we can multiply .....
In addition to illustrating how our formulas work, it provides .....
Besides being very involved, this proof gives no information on .....
This set is obtained **by letting** $n \to \infty$.
It is important to pay attention to domains of definition **when trying** to .....
The following theorem is the key **to constructing** .....
The reason **for preferring** (1) to (2) is simply that .....

3. In certain expressions with "of":

The **idea of combining** (2) and (3) came from .....
The **problem** considered there was that **of determining** WF(u) for .....
We use the **technique of extending** .....

This method has the **disadvantage of**
- **being** very involved.
- **requiring** that f be positive. [Note the infinitive.]

Actually, S has the much stronger **property of being** convex.

4. After certain verbs, especially with prepositions:

We **begin by analyzing** (3).
We **succeeded** ⟨were successful⟩ **in proving** (4).
[*Not*: "succeeded to prove"]
We next **turn to estimating** .....
They **persisted in investigating** the case .....
We are **interested in finding** a solution of .....
We were **surprised at finding out** that .....
[*Or*: surprised to find out]
Their study **resulted in proving** the conjecture for .....
The success of our method will **depend on proving** that .....
To compute the norm of ..... **amounts to finding** .....
We should **avoid using** (2) here, since .....
[*Not*: "avoid to use"]
We **put off discussing** this problem to Section 5.
It **is worth noting** that ..... [*Not*: "worth to note"]
It is worth while discussing here this phenomenon.
[*Or*: worth while to discuss; "worth while" with ing-forms is best avoided as it often leads to errors.]
It is an idea **worth carrying out**.
[*Not*: "worth while carrying out", *nor*: "worth to carry out"]
After **having finished proving** (2), we will turn to .....
[*Not*: "finished to prove"]
(2) **needs handling** with greater care.
One more case **merits mentioning** here.
In [7] he **mentions having proved** this for f not in S.

5. Present Participle in a separate clause (note that the subjects of the main clause and the subordinate clause must be the same):

We show that f satisfies (2), thus **completing** the analogy with .....
Restricting this to R, **we** can define .....
[*Not*: "Restricting ....., the lemma follows". The lemma does not restrict!]
The set A, **being** the union of two continua, is connected.

6. Present Participle describing a noun:

We need only consider paths **starting** at 0.
We interpret f as a function with image **having** support in .....
We regard f as **being** defined on .....

7. In expressions which can be rephrased using "where" or "since":

J is defined to equal Af, the function f **being** as in (3).
[= where f is .....]
This is a special case of (4), the space X here **being** $B(K)$.
We construct 3 maps of the form (5), each of them **satisfying** (8).
....., the limit **being assumed** to exist for every x.

The ideal is defined by $m = \dots$, it **being understood** that .....
F **being** continuous, we can assume that ..... [= Since F is .....]
..... (it **being** impossible to make A and B intersect)
[= since it is impossible]

[Do not write "a function being an element of X" if you mean "a function which is an element of X".]

8. In expressions which can be rephrased as "the fact that X is .....":

Note that M **being** cyclic implies F is cyclic.
The probability of X **being** rational equals 1/2.
In addition to f **being** convex, we require that .....

PASSIVE VOICE

1. Usual passive voice:

This theorem was proved by Milnor in 1976.

In items 2–6, passive voice structures replace sentences with subject "we" or impersonal constructions of other languages.

2. Replacing the structure "we do something":

This identity **is established** by observing that .....
This difficulty **is avoided** above.
When this **is substituted** in (3), an analogous description of K is obtained.
Nothing **is assumed** concerning the expectation of X.

3. Replacing the structure "we prove that X is":

M | is easily **shown to have** .....
M | may **be said to be** regular if .....

This equation **is known to hold** for .....

4. Replacing the construction "we give an object X a structure Y":

Note that E **can be given a** complex **structure** by .....
The letter A is here **given a bar** to indicate that .....

5. Replacing the structure "we act on something":

This order behaves well when g **is acted upon** by an operator.
F **can be thought of** as .....
So all the terms of (5) **are accounted for**.
This case **is met with** in diffraction problems.
The preceding observation, when **looked at** from a more general point of view, leads to .....
In the physical context already **referred to**, K is .....

6. Meaning "which will be (proved etc.)":

Before stating the result **to be proved**, we give .....
This is a special case of convolutions **to be introduced** in Chapter 8.
We conclude with two simple lemmas **to be used** mainly in .....

QUANTIFIERS

This implies that A contains | **all** open subsets of U.
| **all** y with $Gy = 1$.

Let B be the collection of | **all** transforms F of the form .....
| **all** A such that .....

F is defined at **all** points of X.
for **all** $n \neq 0$; for **all** m which have .....; for **all** other m;
for **all but** a finite number of indices i

X contains **all the** boundary except the origin.
The integral is taken over **all of** X.

E, F and G | **all** extend to a neighbourhood of U.
| **all** have their supports in U.
| are **all** zero at x.
| are **all** equal.

There exist functions R, **all of whose** poles are in U, with .....
Each of the following 9 conditions implies **all the others**.
Such an x exists iff **all the** intervals A_x have .....

For **every** g in X ⟨not in X⟩ there exists an N .....
[*But*: for all f and g, for any two maps f and g; "every" is followed by a *singular* noun.]
To **every** f there corresponds a unique g such that .....
Every invariant subspace of X is of the form .....
[Do not write: "Every subspace is not of the form ....." if you mean: "No subspace is of the form ....."; "every" must be followed by an *affirmative* statement.]
Thus $f \neq 0$ at **almost every** point of X.

Since $A_n = 0$ for **each** n, ... [Each = every, considered separately]
Each term in this series is either 0 or 1.
F is bounded on **each** bounded set.
Each of these four integrals is finite.

These curves arise from ....., and **each** consists of .....
There remain four intervals of length 1/16 **each**.
X assumes values $0, 1, \ldots, 9$, **each** with probability 1/10.
$F_1, \ldots, F_n$ are **each** defined in the interval $[0, 1]$.

Those n disjoint boxes are translates **of each other**.

If K is now **any** compact subset of H, there exists .....
[Any = whatever you like; write "for all x", "for every x" if you just mean a quantifier.]
Every measure can be completed, so whenever it is convenient, we may assume that **any** given measure is complete.

There is a subsequence such that .....
There exists an x with .....
[*Or*: there exists x, *but*: there is an x]
There are sets satisfying (2) but not (3).
There is only one such f.
There is a unique function f such that .....
Each f lies in zA for **some** A ⟨at least one A/ exactly one A/at most one A⟩.
Note that **some of** the X_n may be repeated.

F has **no** fixed vector ⟨**no** pole⟩ in U. [*Or*: no poles]
F has **no** limit point in U (hence **none** in K).
Call a set dense if its closure contains **no** nonempty open subset.
If **no two** members of A have an element in common, then .....
No two of the spaces X, Y, and Z are isomorphic.
It can be seen that **no** x has more than one inverse.
In other words, for **no** real x does $\lim F_n(x)$ exist.
[Note the inversion after the negative clause.]
If there is **no** bounded functional such that .....

..... provided **none of** the sums is of the form .....
Let A_n be a sequence of positive integers **none of which** is one less than a power of two.
If there is an f such that ....., we put ..... If **there are** ⟨is⟩ **none**, we define .....
None of these are ⟨is⟩ possible.

Both f and g are obtained by .....
[*Or*: f and g are both obtained]
For **both** C^∞ and analytical categories, ...
C behaves covariantly with respect to maps of **both** X and G.
We now apply (3) to **both** sides of (4).
Both ⟨**these/the**⟩ conditions are restrictions only on .....
[*Note*: "the" after "both"]
C lies on no segment **both of whose** endpoints are in K.
Two consecutive elements do not belong **both to** A *or* **both to** B.
Both its sides are convex. [*Or*: Its sides are both convex.]
B and C are positive numbers, not **both** 0.
Choose points x in M and y in N, **both** close to z, and .....
We show how this method works in 2 cases. In **both**, C is .....

In **either** case, it is clear that ..... [= In both cases]
Each f can be expressed in **either of** the forms (1) and (2).
[= in any of the two forms]
The density of $X + Y$ is given by **either of** the two integrals.
The two classes coincide if X is compact. In that case we write $C(X)$ for **either of** them.
Either f or g must be bounded.

Let u and v be two distributions **neither of** which is .....
[Use "neither" when there are *two* alternatives.]
This is true for **neither of** the two functions.
Neither statement is true.
In **neither** case can f be smooth.
[Note the inversion after a negative clause.]
He proposes two conditions, but **neither** is satisfactory.

NUMBER, QUANTITY, SIZE

1. Cardinal numbers:

A and B are also F-functions, any **two** of A, B, and C being independent.

the multi-index with | all entries **zero** except the kth which is **one**
| the last k entries **zero**

This shows that there are no **two** points a and b such that .....
There are **three** that the reader must remember. [= three of them]
We have defined A, B, and C, and **the three** sets satisfy .....
For **the two** maps defined in Section 3, ...
["The" if only two maps are defined there.]
R is concentrated at **the** n points $x_1, \dots, x_n$ defined above.

for **at least** ⟨**at most**⟩ one k; with norm **at least equal to** 2

There are **at most 2 such** r in $(0, 1)$.
There is **a unique** map satisfying (4).
(4) has **a unique** solution g for each f.
But: (4) has **the unique** solution $g = ABf$.
(4) has **one and only one** solution.
Precisely r **of** the intervals are closed.
In Example 3 only **one of** the x_j is positive.
If $p = 0$ then there are **an additional** m arcs.

2. Ordinal numbers:

The first two are simpler than **the third**.
Let S_i be **the first of** the remaining S_j.
The n**th** trial is the last.
X_1 appears at **the** $(k + 1)$**th** place.

The gain **up to and including the** n**th** trial is .....
The elements of **the third and fourth** rows are in I.
[Note the plural.]
F has a zero of **at least third** order at x.

3. Fractions:

Two-thirds of its diameter is covered by .....
But: **Two-thirds** of the gamblers are ruined.
G is **half** the sum of the positive roots.
[*Note*: Only "half" can be used with or without "of".]
On the average, about **half** the list will be tested.
J contains an interval of **half** its length in which .....

F is greater by **a half** ⟨a third⟩.
The other player is half ⟨**one third**⟩ as fast.
We divide J **in half**.
All sides were increased by the same **proportion**.
About **40 percent** of the energy is dissipated.
A positive **percentage** of summands occurs in all the k partitions.

4. Smaller ⟨greater⟩ than:

n is | **greater** ⟨less⟩ **than** k.
| **much** ⟨substantially⟩ **greater** than k.
| **no greater** ⟨smaller⟩ **than** k.
| **greater** ⟨less⟩ **than or equal to** k.
| [*Not*: "greater or equal to"]
| **strictly less than** k.

All points at a distance **less than** K from A satisfy (2).
We thus obtain a graph of **no more than** k edges.

This set has | **fewer** elements than K has.
| **no fewer than** twenty elements.

F can have no jumps **exceeding** 1/4.
The degree of P **exceeds** that of Q.
Find the density of **the smaller of** X and Y.
The smaller of the two satisfies .....
F is dominated ⟨**bounded**/estimated/majorized⟩ by .....

5. How much smaller ⟨greater⟩:

25 is **3 greater** than 22. 22 is **3 less** than 25.
Let a_n be a sequence of positive integers none of which is **one less** than a power of two.
The degree of P **exceeds** that of Q **by at least** 2.
f is **greater by a half** ⟨a third⟩.
C is **less than a third** of the distance between .....

Within I, the function f **varies** ⟨oscillates⟩ **by less than** l.
The upper and lower limits of f **differ by at most** 1.
We thus have in A **one** element **too many**.
On applying this argument k **more times**, we obtain .....
This method is recently **less and less** used.
A succession of **more and more** refined discrete models.

6. How many times as great:

twice ⟨**ten times**/one third⟩ **as long as**; half as big as
The longest edge is at most 10 times as long as the shortest one.
A has **twice as many** elements as B has.
J contains a subinterval **of half its length** in which .....
A has four times the radius of B.
The diameter of L is $1/k$ times ⟨**twice**⟩ **that of** M.

7. Multiples:

The k-**fold** integration by parts shows that .....
F covers M **twofold**.

M is bounded by **a multiple of** t ⟨a constant times t⟩.
This distance is less than **a** constant **multiple of** d.
G acts on H as **a multiple**, say n, of V.

8. Most, least, greatest, smallest:

F has **the most** ⟨the fewest⟩ points when .....
In **most** cases it turns out that .....
Most of the theorems presented here are original.

The proofs are, **for the most part**, only sketched.
Most probably, his method will prove useful in .....
What **most** interests us is whether .. ..

The **least** such constant is called the norm of f.
This is **the least** useful of the four theorems.
The method described above seems to be **the least** complex.
That is **the least** one can expect.
The elements of A are comparatively big, but **least** in number.
None of those proofs is easy, and John's **least of all**.

The best estimator is a linear combination U such that var U is **smallest possible**.
The expected waiting time is **smallest** if .....
L is **the smallest number** such that .....
F has **the smallest** norm among all f such that .....
K is **the largest of** the functions which occur in (3).
There exists **a smallest** algebra with this property.
Find the **second largest** element in the list L.

9. Many, few, a number of:

There are [Note the plural.]
- **a large number of** illustrations.
- only **a finite number of** f with $Lf = 1$.
- **a small number** of exceptions.
- **an infinite number of** sets .....
- **a negligible number of** points with .....

Ind c is **the number of times** that c winds around 0.
We give **a number of** results concerning ..... [= some]
This may happen in **a number of** cases.
They correspond to the values of **a countable number of** invariants.
..... for **all** n **except a finite number** ⟨for **all but finitely many** n⟩.
Q contains **all but a countable number** of the f^i.

There are only **countably many** elements q of Q with dom $q = S$.

The theorem is fairly general. There are, however, **numerous** exceptions.
A variety of other characteristic functions can be constructed in this way.

There are **few** exceptions to this rule. [= not many]
Few of various existing proofs are constructive.
He accounts for all the major achievements in topology over **the last few** years.
The generally accepted point of view in this domain of science seems to be changing **every few** years.
There are **a few** exceptions to this rule. [= some]
Many interesting examples are known. We now describe **a few of** these.
Only **a few of** those results have been published before.
Quite a few of them are now widely used. [= A considerable number]

10. Equality, difference:

A equals B *or* A is equal to B [*Not*: "A is equal B"]

The Laplacian of g is $4r > 0$.
The inverse of FG is GF.
Then r is about kn.
The norms of f and g coincide.

F has the same number of zeros and poles in U.

F and G **differ by** a linear term ⟨by a scale factor⟩.
The differential of f is **different from** 0.
Each member of G **other than** the identity mapping is .....
F is not identically 0.
Let a, b and c be **distinct** complex numbers.
Each w is Pz for precisely m distinct values of z.

Functions which are equal a.e. are indistinguishable as far as integration is concerned.

11. Numbering:

Exercises 2 to 5 furnish other applications of this technique. [*Amer.*: Exercises 2 through 5]

in the third and fourth rows
from line 16 onwards
in lines 16–19
the next-to-last column
the last paragraph but one of the previous proof
the derivatives up to order k
the odd-numbered terms

The matrix with | 1 in the (i, j) entry and zero elsewhere
| all entries zero except for $N - j$ at (N, j)

This is | hinted at in Sections 1 and 2.
| quoted on page 36 of [4].

HOW TO AVOID REPETITION

1. Repetition of nouns:

Note that the continuity of f implies **that** of g.
The passage from Riemann's theory to **that** of Lebesgue is .....
The diameter of F is about twice **that** of G.
His method is similar to **that** used in our previous paper.
The nature of this singularity is the same as **that** which f has at $x = 0$.
Our results do not follow from **those** obtained by Lax.

One can check that the metric on T is **the one** we have just described.
It follows that S is the union of two disks. Let D be **the one** that contains .....
The cases $p = 1$ and $p = 2$ will be **the ones** of interest to us.
We prove a uniqueness result, similar to **those** of the preceding section.

Each of the functions on the right of (2) is **one** to which .....
F has many points of continuity. Suppose x is **one**.
In addition to a contribution to W_1, there may be **one** to W_2.

We now prove that the constant pq cannot be replaced by a smaller **one**.
Consider the differences between these integrals and **the** corresponding **ones** with f in place of g.
The geodesics (4) are **the** only **ones** that realize the distance between their endpoints.
On account of the estimate (2) and similar **ones** which can be .....

We may replace A and B by whichever is the larger of **the two**. [*Not*: "the two ones"]
This inequality applies to conditional expectations as well as to ordinary **ones**.

One has to examine the equations (4). If **these** have no solutions, then .....
D yields operators D^+ and D^-. **These** are formal adjoints of each other.
This gives rise to the maps F_i. All the other maps are suspensions of **these**.
F is the sum of A, B, C and D. The last two of **these** are zero.

Both f and g are connected, but **the latter** is in addition compact. [The latter = the second of *two* objects]
Both AF and BF were first considered by Banach, but only **the former** is referred to as the Banach map, **the latter** being called the Hausdorff map.
We have thus proved Theorems 1 and 2, **the latter** without using .....

Since the vectors G_i are orthogonal to **this last** space, ...
As a consequence of **this last** result, ...
Let us consider sets of the type (1), (2), (3) and (4). **These last two** are called .....

We shall now describe a general situation in which **the last-mentioned** functionals occur naturally.

2. Repetition of adjectives, adverbs or phrases like "x is .....":

If f and g are measurable functions, then **so are** $f + g$ and $f \cdot g$.
The union of measurable sets is a measurable set; **so is** the complement of every measurable set.
The group G is compact and **so is** its image under f.
It is of the same fundamental importance in analysis **as is** the construction of .....

F is bounded but **is not** necessarily **so** after division by G.

Show that there are many **such** Y.
There is only one **such** series for each y.
Such an h is obtained by .....

3. Repetition of verbs:

A geodesic which meets bM **does so** either transversally or .....
This will hold for $x > 0$ if it **does** for $x = 0$.
Note that we have not required that ....., and we shall not **do so** except when explicitly stated.
The integral might not converge, but it **does so** after .....

We will show below that the wave equation can be put in this form, **as can** many other systems of equations.
The elements of L are not in S, **as** they **are** in the proof of .....

4. Repetition of whole sentences:

The same is true for f in place of g.
The same being true for f, we can ..... [= Since the same .....]
The same holds for ⟨applies to⟩ the adjoint map.

We shall assume that **this is the case**.
Such was the case in (2).
The L^2 theory has more symmetry **than is the case** in L^1.
Then either ..... or ..... **In the latter** ⟨former⟩ **case**, ...

For k **this is no longer true**.
This is not true of (2).

This is not so in other queuing processes.
If **this is so**, we may add .....
If $f_i \in L$ and if $F = f_1 + \ldots + f_n$ then $F \in H$, and every F is **so** obtained.

We would like to ..... If U is open, **this** can be done.
On S, **this** gives the ordinary topology of the plane.
Note that **this** is not equivalent to .....
[Note the difference between "this" and "it": you say "*it* is not equivalent to" if you are referring to some object explicitly mentioned in the preceding sentence.]

F **has the stated** ⟨desired/claimed⟩ **properties**.

WORD ORDER

General remarks: The normal order is: subject + verb + direct object + adverbs in the order manner-place-time.
Adverbial clauses can also be placed at the beginning of a sentence, and some adverbs always come between subject and verb. Subject almost always precedes verb, except in questions and some negative clauses.

1. ADVERBS

1a. Between subject and verb, but after "be"; in compound tenses after first auxiliary

• *Frequency adverbs*:

This has **already** been proved in Section 8.
This result will **now** be derived computationally.
Every measurable subset of X is **again** a measure space.
We **first** prove a reduced form of the theorem.
There has **since** been little systematic work on .....
It has **recently** been pointed out by Fix that .....
It is **sometimes** difficult to .....
This **usually** implies further conclusions about f.
It **often** does not matter whether .....

• *Adverbs like "also", "therefore", "thus"*:

Our presentation is **therefore** organized in such a way that .....
The sum in (2), though formally infinite, is **therefore** actually finite.
One must **therefore** also introduce the class of .....
C is connected and is **therefore** not the union of .....

These properties, with the exception of (1), **also** hold for t.
We will **also** leave to the reader the verification that .....
It will **thus** be sufficient to prove that .....
(2) implies (3), since one would **otherwise** obtain $k = 0$.

The order of several topics has **accordingly** been changed.

• *Emphatic adverbs* (*clearly*, *obviously*, *etc.*):

It would **clearly** have been sufficient to assume that .....
F is **clearly** not an I-set.
Its restriction to N is **obviously** just f.
This case must **of course** be excluded.
The theorem **evidently** also holds if $x = 0$.

The crucial assumption is that the past history **in no way** influences .....
We did not **really** have to use the existence of T.
The problem is to decide whether (2) **really** follows from (1).
The proof is now **easily** completed.
The maximum is **actually** attained at some point of M.

We then **actually** have ..... [= We have even more]
At present we will **merely** show that .....
A stronger result is **in fact** true.
Throughout integration theory, one **inevitably** encounters ∞.
But H itself can **equally well** be a member of S.

1b. After verb—most adverbs of manner:

We conclude **similarly** that .....
One sees **immediately** that .....
Much relevant information can be obtained **directly** from (3).
This difficulty disappears **entirely** if .....
This method was used **implicitly** in random walks.

1c. After an object if it is short:

We will prove the theorem **directly** without using the lemma.
But: We will prove **directly** a theorem stating that .....
This is true for every sequence that shrinks to x **nicely**.
Define Fg **analogously** as the limit of .....
(2) defines g **unambiguously** for every g'.

1d. At the beginning—adverbs referring to the whole sentence:

Incidentally, we have now constructed .....
Actually, Theorem 3 gives more, namely .....
Finally, (2) shows that $f = g$. [*Not*: "At last"]
Nevertheless, it turns out that .....
Next, let V be the vector space of .....

More precisely, Q consists of .....
Explicitly ⟨**Intuitively**⟩, this means that .....
Needless to say, the boundedness of f was assumed only for simplicity.
Accordingly, either f is asymptotically dense or .....

1e. In front of adjectives—adverbs describing them:

a **slowly varying** function
probabilistically significant problems
a method **better suited** for dealing with .....

F and G are **similarly obtained** from H.
F has a **rectangularly shaped** graph.
Three-quarters of this area is covered by **subsequently chosen** cubes. [Note the singular.]

1f. "only"

We need the openness **only** to prove the following.
It reduces to the statement that **only** for the distribution F do the maps F_i satisfy (2). [Note the inversion.]
In this chapter we will be concerned **only** with .....

In (3) the X_j assume the values 0 and 1 **only**.
If (iii) is required for finite unions **only**, then .....

We need **only** require (5) to hold for bounded sets.
The proof of (2) is similar, and will **only** be indicated briefly.
To prove (3), it **only** remains to verify .....

2. ADVERBIAL CLAUSES

2a. At the beginning:

In testing the character of ....., it is sometimes difficult to .....
For $n = 1, 2, \ldots$, consider a family of .....

2b. At the end (normal position):

The averages of F_n become small **in small neighbourhoods** of x.

2c. Between subject and verb, but after first auxiliary—only short clauses:

The observed values of X will **on the average** cluster around .....
This could **in principle** imply an advantage.
For simplicity, we will **for the time being** accept as F only C^2 maps.

Accordingly we are **in effect** dealing with .....
The knowledge of f is **at best** equivalent to .....
The stronger result is **in fact** true.
It is **in all respects** similar to matrix multiplication.

2d. Between verb and object if the latter is long:

It suffices **for our purposes** to assume .....
To a given density on the line there corresponds **on the circle** the density given by .....

3. INVERSION AND OTHER PECULIARITIES

3a. Adjective or past participle after a noun:

Let Y be the complex X with the origin **removed**.
Theorems 1 and 2 **combined** give a theorem .....
We now show that G is in the symbol class **indicated**.
We conclude by the part of the theorem **already proved** that .....
The bilinear form **so defined** extends to .....
Then for A **sufficiently small** we have .....
By queue length we mean the number of customers **present** including the customer **being served**.
The description is the same with the roles of A and B **reversed**.

3b. Direct object or adjectival clause placed farther than usual—when they are long:

We must **add** to the right side of (3) **the probability** that .....
This is equivalent to **defining** in the z-plane **a density** with .....
Denote for the moment by f **the element** satisfying .....
F is the **restriction** to D **of** the unique linear map .....
The **probability** at birth **of** a lifetime exceeding t is at most .....

3c. Inversion in some negative clauses:

We do not assume that ....., **nor do** we assume a priori that .....
Neither is the problem simplified by assuming $f = g$.
The "if" part now follows from (3), since **at no point can** S exceed the larger of X and Y.
The fact that **for no** x **does** Fx contain y implies that .....

In no case does the absence of a reference imply any claim to originality on my part.

3d. Inversion—other examples:

F is compact and **so is** G.
If f, g are measurable, then **so are** $f + g$ and $f \cdot g$.

Only for $f = 1$ | **can** one expect to obtain .....
| **does** that limit exist.

3e. Adjective in front of "be"—for emphasis:

By far **the most important** is the case where .....
Much more subtle are the following results of John.
Essential to the proof are certain topological properties of M.

3f. Subject coming sooner than in some other languages:

Equality occurs in (1) iff f is constant.
The natural **question** arises whether it is possible to .....
In the following applications **use** will be made of .....
Recently **proofs** have been constructed which use .....

3g. Incomplete clause at the beginning or end of a sentence:

Put differently, the moments of arrival of the lucky customers constitute a renewal process.
Rather than discuss this in full generality, let us look at .....
It is important that the tails of F and G are of comparable magnitude, **a statement** made more precise by the following inequalities.

WHERE TO INSERT A COMMA

General rules: Do not over-use commas—English usage requires them less often than in many other languages. Do not use commas around a clause that defines (limits, makes more precise) some part of a sentence. Put commas before and after non-defining clauses (i.e. ones which can be left out without damage to the sense). Put a comma where its lack may lead to ambiguity, e.g. between two symbols.

1. Comma not required:

We shall now prove that f is proper.
The fact that f has radial limits was proved in [4].
It is reasonable to ask whether this holds for $g = 1$.
M is the set of all maps which take values in V.
There is a polynomial P such that $Pf = g$.
The element given by (3) is of the form (5).
Let M be the manifold to whose boundary f maps K.
Take an element all of whose powers are in S.

F is called proper if G is dense.
There exists a D such that $DxyH$ whenever $HxyG$.

$F(x) = G(x)$ for all $x \in X$.
Let F be a nontrivial continuous linear operator in V.

2. Comma required:

The proof of (3) depends on the notion of M-space, which has already been used in [4].
We will use the map H, which has all the properties required.
There is only one such f, and (4) defines a map from .....

In fact, we can do even better.
In this section, however, we will not use it explicitly.
Moreover, F is countably additive.
Finally, (d) and (e) are consequences of (4).
Nevertheless, he succeeded in proving that .....
Conversely, suppose that .....
Consequently, (2) takes the form .....
In particular, f also satisfies (1).

Guidance is also given, whenever necessary or helpful, on further reading.
This observation, when looked at from a more general point of view, leads to .....
It follows that f, being convex, cannot satisfy (3).
If $e = 1$, which we may assume, then .....
We can assume, by decreasing k if necessary, that .....
Then (5) shows, by Fubini's theorem, that .....

Put this way, the question is not precise enough.
Being open, V is a union of disjoint boxes.

This is a special case of (4), the space X here being $B(K)$.

In [2], X is assumed to be compact.
For all x, $G(x)$ is convex.
[Comma between two symbols.]
In the context already referred to, K is the complex field.
[Comma to avoid ambiguity.]

3. Comma optional:

By Theorem 2, there exists an h such that .....
For z near 0, we have .....
If h is smooth, then M is compact.
Since h is smooth, M is compact.
It is possible to use (4) here, but it seems preferable to .....
This gives (3), because ⟨since⟩ we may assume .....

Integrating by parts, we obtain .....
To do this, put .....

X, Y, and Z are compact.
$X = FG$, where F is defined by .....
Thus ⟨Hence/Therefore⟩, we have .....

SOME TYPICAL ERRORS

1. **Spelling errors:**

Spelling should be consistent, either British or American throughout:

Br.: colour, neighbour, centre, fibre, labelled, modelling
Amer.: color, neighbor, center, fiber, labeled, modeling

an unified approach ⇝ a unified approach
a M such that ⇝ an M such that
[Use "a" or "an" according to pronunciation.]

2. **Grammatical errors:**

Let f denotes ⇝ Let f denote
Most of them is ⇝ Most of them are
There is a finite number of ⇝ There are a finite number of
In 1964 Lax has shown ⇝ In 1964 Lax showed
[Use the past tense if a date is given.]

The Taylor's formula ⇝ Taylor's formula [*Or*: the Taylor formula]
The section 1 ⇝ Section 1
Such map exists ⇝ Such a map exists [*But*: for every such map]
In the case M is compact ⇝ In case M is compact
[*Or*: In the case where M is compact]
In case of smooth norms ⇝ In the case of smooth norms
We are in the position to prove ⇝ We are in a position to prove

F is equal G ⇝ F is equal to G [*Or*: F equals G]
F is greater or equal to G ⇝ F is greater than or equal to G

Continuous in the point x ⇝ Continuous at x
Disjoint with B ⇝ Disjoint from B
Equivalent with B ⇝ Equivalent to B
Independent on B ⇝ Independent of B
[*But*: depending on B]
Similar as B ⇝ Similar to B

Similarly as in Sec. 2 ⇝
- Similarly to Sec. 2
- As ⟨Just as⟩ in Sec. 2
- As is the case in Sec. 2
- In much the same way as in Sec. 2

On Fig. 3 ⇝ In Fig. 3
In the end of Sec. 2 ⇝ At the end of Sec. 2

Since $f = 0$ then M is closed ⇝ Since $f = 0$, M is closed
[*Or*: Since $f = 0$, we conclude that M is closed]
....., as it is shown in Sec. 2 ⇝ ....., as is shown in Sec. 2
Every function being an element of X is convex
⇝ Every function which is an element of X is convex
Setting $n = p$, the equation can be .....
⇝ Setting $n = p$, we can ..... [Because *we* set.]

3. **Wrong word used:**

We now give few examples [= not many]
⇝ We now give a few examples [= some]
Summing (2) and (3) by sides ⇝ Summing (2) and (3)
In the first paragraph ⇝ In the first section
....., which proves our thesis
⇝ ....., which proves our assertion ⟨conclusion/statement⟩
[thesis = dissertation]
For n big enough ⇝ For n large enough
To this aim ⇝ To this end
At first, note that ⇝ First, note that
At last, we obtain ⇝ Finally, we obtain
For every two elements ⇝ For any two elements
....., what completes the proof ⇝ ....., which completes the proof
....., what is impossible ⇝ ....., which is impossible

4. **Wrong word order:**

The described above condition ⇝ The condition described above

The both conditions ⇝ Both conditions, Both the conditions
Its both sides ⇝ Both its sides

The three first rows ⇝ The first three rows
The two following sets ⇝ The following two sets

This map we denote by f ⇝ We denote this map by f

5. **Other examples:**

We have ⟨obtain⟩ that B is .....
⇝ We see ⟨conclude/deduce/find/infer⟩ that B is .....
We are done ⇝ The proof is complete.

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