CORRECTION DE LA SERIE 2

Exercice 1 : Etude des fonctions a variables variables réellee

```
>
> readlib(iscont):
> readlib(discont):
> f:=x->1/x;
```

$$f := x \to \frac{1}{x}$$

```
> iscont(f(x),x=-infinity..infinity);
```

$$false$$

```
> discont(f(x),x);
```

$$\{0\}$$

```
> limit(f(x),x=0,right);
```

$$\infty$$

```
> limit(f(x),x=0,left);
```

$$-\infty$$

```
> limit(f(x),x=infinity);
```

$$0$$

```
> limit(f(x),x=-infinity);
```

$$0$$

```
> D(f);
```

$$x \to -\frac{1}{x^2}$$

```
> (D@@2)(f);
```

$$x \to \frac{2}{x^3}$$

```
> diff(f(x),x);
```

$$-\frac{1}{x^2}$$

```
> diff(diff(f(x),x),x);
```

$$\frac{2}{x^3}$$

Exercice 2 : Integration des fonctions réelles

```
> g:=x->1/x;
```

$$g := x \to \frac{1}{x}$$

```
>
> int(f(x),x);
```

$$\ln(x)$$

```
> int(f(x),x=2..3);
```

$$\ln(3) - \ln(2)$$

```
> h:=x->exp(x);
```

$$h := \exp$$

Exercice 3 : Développement limité

```
> taylor(h(x),x);
```

$$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + O(x^6)$$

```
> P:=convert(",polynom);
```

$$P := 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5$$

```
> coeff(P,x,3);
```

$$\frac{1}{6}$$

```
> series(h(x),x,4);
```

$$1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + O(x^4)$$

```
> Q:=convert(",polynom);
```

$$Q := 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

Exercice 4: Equations différentielles

```
> eqd:=diff(y(x),x)-2*y(x)=2*x;
```

$$eqd := \left(\frac{\partial}{\partial x}\mathrm{y}(x)\right) - 2\,\mathrm{y}(x) = 2\,x$$

```
> dsolve(eqd,y(x));
```

$$\mathrm{y}(x) = -x - \frac{1}{2} + \mathbf{e}^{(2x)}\_C1$$

```
> dsolve({eqd,y(0)=0},y(x));
```

$$\mathrm{y}(x) = -x - \frac{1}{2} + \frac{1}{2}\mathbf{e}^{(2x)}$$

```
> syst1:={diff(x(t),t)=x(t)+3*y(t),diff(y(t),t)=x(t)-y(t)};
```

$$syst1 := \{\frac{\partial}{\partial t}\mathrm{y}(t) = \mathrm{x}(t) - \mathrm{y}(t), \frac{\partial}{\partial t}\mathrm{x}(t) = \mathrm{x}(t) + 3\,\mathrm{y}(t)\}$$

```
> dsolve(syst1,{x(t),y(t)});
```

$$\{\mathrm{x}(t) = \frac{1}{4}\_C1\,\mathbf{e}^{(-2t)} + \frac{3}{4}\_C1\,\mathbf{e}^{(2t)} + \frac{3}{4}\_C2\,\mathbf{e}^{(2t)} - \frac{3}{4}\_C2\,\mathbf{e}^{(-2t)},$$

$$\mathrm{y}(t) = \frac{1}{4}\_C1\,\mathbf{e}^{(2t)} - \frac{1}{4}\_C1\,\mathbf{e}^{(-2t)} + \frac{3}{4}\_C2\,\mathbf{e}^{(-2t)} + \frac{1}{4}\_C2\,\mathbf{e}^{(2t)}\}$$

```
> syst2:={diff(x(t),t)=x(t)+3*y(t),diff(y(t),t)=x(t)-y(t),x(0)=0,y
  (0)=1};
```

$$syst2 := \{\frac{\partial}{\partial t}\mathrm{y}(t) = \mathrm{x}(t) - \mathrm{y}(t), \frac{\partial}{\partial t}\mathrm{x}(t) = \mathrm{x}(t) + 3\,\mathrm{y}(t), \mathrm{x}(0) = 0, \mathrm{y}(0) = 1\}$$

```
> sol:=dsolve(syst2,{x(t),y(t)});
```

$$sol := \{\mathrm{x}(t) = \frac{3}{4}\mathbf{e}^{(2t)} - \frac{3}{4}\mathbf{e}^{(-2t)}, \mathrm{y}(t) = \frac{3}{4}\mathbf{e}^{(-2t)} + \frac{1}{4}\mathbf{e}^{(2t)}\}$$

```
> op(1,sol);
```

$$x(t) = \frac{3}{4}\,e^{(2\,t)} - \frac{3}{4}\,e^{(-2\,t)}$$

```
> op(2,sol);
```

$$y(t) = \frac{3}{4}\,e^{(-2\,t)} + \frac{1}{4}\,e^{(2\,t)}$$

```
> subs(sol,x(t));
```

$$\frac{3}{4}\,e^{(2\,t)} - \frac{3}{4}\,e^{(-2\,t)}$$

```
> subs(sol,y(t));
```

$$\frac{3}{4}\,e^{(-2\,t)} + \frac{1}{4}\,e^{(2\,t)}$$

```
> eqd:=diff(y(x),x)-2*y(x)=2*x;
```

$$eqd := \left(\frac{\partial}{\partial x}\,y(x)\right) - 2\,y(x) = 2\,x$$

```
> x_n:=dsolve({eqd,y(0)=0},y(x),numeric);
```

$$x\_n := \mathbf{proc}(rkf45\_x)\ \ldots\ \mathbf{end}$$

```
> x_n(3);
```

$$[x = 3,\ y(x) = 198.2143915666270]$$

```
>
```